Proceedings of the I·R·E

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MARCH, 1944

VOLUME 32 NUMBER 3

Radio Progress During 1943

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Centimeter-Band Magnetrons

Direct-Voltage Paper Capacitors

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Increasingly there is need for close understanding and co-operation between two major groups in the radio-and-electronic industry: the commercial leaders and the engineering planners. The PROCEEDINGS OF THE I.R.E. offers its readers expressions of the viewpoints of members of the former group in the form of "guest editorials" which, it is felt, will be stimulating to the engineers and helpful to the industry. Accordingly there is here presented in the form in which it was received an analysis prepared for the PROCEEDINGS by the Vice-President of the Westinghouse Electric and Manufacturing Company—himself an outstanding and pioneer radio worker.

The Editor

Postwar Applications of Wartime Engineering

WALTER EVANS

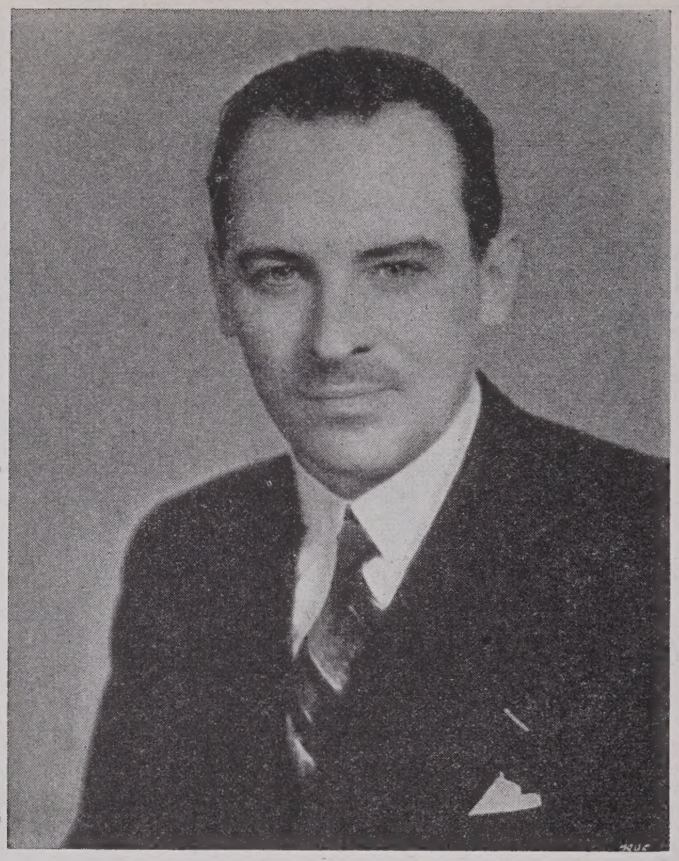
Radio engineers of our country are now veterans of two wars. Their achievements in World War I are well known to all, while their contributions made during the present conflict will be many times more spectacular and far-reaching.

In the first World War, we heard the swan song of the spark transmitter and its companion crystal detector upon which communications in the preceding years had been based. The stimulation of military necessities brought into general use the three-element tube, and its associated technique.

In the entire history of science, there has very likely been no parallel to the effort which has been applied to the use of radio during the present war for communications, ordnance, aids to aircraft, and other military applications. The laboratories of the Government, private individuals, and of the commercial companies have been vastly enlarged, and the contributions which the engineers are making to the success of the effort of the allied nations are truly magnificent.

In all of this work, however, the radio engineer is incurring an obligation to his fellow men and to our civilization in the forthcoming years after victory is finally achieved. Out of the developments of the first war grew new and virile industries: radio broadcasting, the home-receiver field, sound movies, and some industrial applications. With the termination of the present conflict, the radio engineer will be equipped with the knowledge, technique, and even the facilities for industries yet unborn. The contribution of the radio engineer has resulted in vast manufacturing facilities; it becomes in a measure his obligation to keep those filled, and to supply useful work for the many men returning with technical training from the armed forces.

This proposes a new and difficult problem. The engineer normally is supplied with the specifications of the result to be achieved, and works toward that end. In this case, he has the know-how and facilities or, one might say, he has the results, but must find the gainful use to which it can be put. It calls for a reversal of viewpoint and a brand of ingenuity found chiefly among the engineering and scientific talent of our country. That they will find such applications from which new industries will be developed, I have no doubt.



E. M. Deloraine

E. M. Deloraine, general director of the laboratories division of Federal Telephone and Radio Corporation, manufacturing affiliate of International Telephone and Telegraph Corporation, was elected a director at a meeting on February 2, 1944, of the I. T. & T. Board of Directors.

Mr. Deloraine, who has been closely associated with research and development activities of his organization since 1925, was born in Paris, France. Developing an interest in science at an early age, he received his first training in research in L'École de Physique et Chimie, a branch of Paris University. He was interrupted in his studies in 1917 to join the French Army Signal Corps, and after the Armistice engaged in research work at the Eiffel Tower under General Ferrié, head of the French Signal Corps. He returned to college to complete his studies and, upon graduation as a physicist in 1920 with highest honors, continued his work at the Eiffel Tower station. A year later he joined the London engineering staff of the International Western Electric Company, later the International Standard Electric Company, under Sir Frank Gill, and began technical work in connection with broadcasting at the experimental station 2WP. Until 1925 he was responsible for part of the developments in Great Britain in connection with the first transatlantic telephone circuit. He was made European technical director of this latter company in 1933.

Mr. Deloraine was for a time actively in charge of developments which brought about the establishment of the first Madrid-Buenos Aires radiotelephone circuit. Under his direction in 1931 the first demonstrations of single-sideband short-wave radiotelephone were carried out between Buenos Aires and Madrid, and between Madrid and Paris, establishing the well-recognized improvements in transmission efficiency and economies made possible by this method.

In 1929 he demonstrated long-distance telephone communication to ships at sea, conducting for the first time telephone conversations with the S.S. *Berengaria* in mid-ocean.

It was in the years following that Mr. Deloraine made some of his most important contributions in the development of ultra-high frequencies. In 1931 and 1933 he established telephone and printer communications across the English Channel on approximately 1700 megacycles, using very sharp beams, and in 1936 and 1937 made possible the first multichannel ultra-short-wave telephone link. Later he used ultra-high frequency in connection with television transmission, including the construction of the station at the Eiffel Tower, providing the highest power used.

He was also active in the advancement of high-power broadcasting. As early as 1932 he established the Prague Station with 120 kilowatts carrier, followed two years later by the Budapest Station with the same carrier power and unique for its antifading mast antenna, over 1000 feet high, the highest antenna ever constructed. In 1939 he made a proposal to the French Post and Telegraph Administration for a high-frequency broadcasting center of twelve stations of 150 kilowatts carrier each. This project was adopted.

Mr. Deloraine was successful in directing experiments in connection with automatic radio compasses for aircraft. This technique was demonstrated in the United States for the first time in 1937. He came to this country in 1941 to take charge of the organization of the laboratories unit for Federal Telephone and Radio Corporation.

Mr. Deloraine was made a Chevalier of the Legion of Honor in 1938 for exceptional services to the Post and Telegraph Department of France, and he was elected Vice-President of the French Institute of Radio Engineers in 1939. He has been a member of the International Consultative Committee of Long Distance Telephony since 1927. He is also a member of the French and Belgian Societies of Electricians and the French Astronomical Society. He became a member of the Institute of Radio Engineers in 1940 and in 1941 was awarded the grade of Fellow.

Radio Progress During 1943*

Introduction

WAR requirements continued to dominate radio during 1943, particularly in the fields of engineering and manufacturing. Standardization of specifications for component parts contributed greatly to the interchangeability of units and facilitated the quantity manufacture of preferred types.

Public announcement was made of the extensive use of radio methods for the detection and location of distant objects but there has been no publication of the technical means employed.

Transmitters and Antennas

General

In the fields of standard broadcasting, high-frequency broadcasting, facsimile broadcasting, and television, efforts were limited substantially to the maintenance of existing radio plant.

Antennas

Noteworthy contributions were published on the subject of antennas. One summarized progress in aircraft loop antennas used for reception and direction finding. It reported that low-impedance loops are used in preference to the high-impedance type on transport and military aircraft because of difficulties in hermetically sealing such loops and because of the problem of constructing and installing low-loss and low-capacitance transmission lines. To obtain the required null for direction-finding purposes, the antenna effect, resulting from the electrostatic component of a wave, must be overcome by the use of the tubular gap-type electrostatic shield. This shield also eliminates certain kinds of electrical noises and reduces characteristic aircraft precipitation static. The loop is normally located on top of or beneath the fuselage to minimize quadrantal errors caused by reflection or refraction of waves by the wings or fuselage. Iron-core loop antennas have been used quite extensively, although not in the United States. The iron core increases the Q which in turn increases the pickup. This type of core permits the use of fewer turns, smaller size, and improved aerodynamic design, the latter because the loop may be closer to the surface of the aircraft without lowering the Q.

(1) G. F. Levy, "Loop antennas for aircraft," PROC. I.R.E., vol. 31, pp. 56–66; February, 1943.

The definition for the "effective length" of a transmitting antenna, recently published, was used in deriving expressions for the radiation function, radiation resistance, directivity, and gain, of a two-antenna array (vertical) when the relative phase of excitation and current amplitudes are of arbitrary value.

(2) C. W. Harrison, Jr., "A note on the characteristics of the two-antenna array," PROC. I.R.E., vol. 31, pp. 75–78; February, 1943.

An improved calculating machine was reported. It permits the calculation of the polar diagram produced by as many as five antennas situated anywhere within a circle of four wavelengths' diameter. The currents in the antennas may have any relative phases and magnitudes. The machine has been in constant service for about two years.

(3) H. P. Williams, "A machine for calculating the polar diagram of an antenna system," *Elec. Commun.*, vol. 21, pp. 103–111; 1943.

The theoretical optimum-current distribution on a vertical antenna of given length was defined as that current distribution giving the maximum possible field strength on the horizon for a given power output. The problem of determining such distributions was set up as a problem in the calculus of variations, and solution functions are derived for antennas varying in length from one eighth of a wavelength up to a full wavelength.

It was shown that the apparent antenna performance obtained with the theoretical optimum distribution was as good as or better than that obtained with any practical distribution, and thus served to bound the improvement in antenna performance which may be expected as a result of changes in current distribution. A curve of maximum possible field strength on the horizon for fixed power output versus antenna height was given.

Finally, these theoretical optimum-current distributions were used to indicate the general class of distributions most likely to yield worth-while results in a search for practical optimum distributions. Several such practical distributions were considered in detail.

(4) L. LaPaz and G. A. Miller, "Optimum current distributions on vertical antennas," PROC. I.R.E., vol. 31, pp. 214–232; May, 1943.

An expression was derived for the mutual impedance of a symmetrical center-driven antenna in proximity to an untuned parasitic element, when the wires are parallel and are not displaced in length. An integral frequently occurring in antenna problems was evaluated graphically over the range required in this analysis.

(5) C. W. Harrison, Jr., "A note on the mutual impedance of antennas," *Jour. Appl. Phys.*, vol. 14, pp. 306–309; June, 1943.

The cylindrical center-driven antenna was analyzed as a boundary-value problem of electromagnetic theory. An integral equation in the current (originally obtained in a different way by Hallén) was derived. Its solution was outlined briefly and the general formula was given. Complete curves for the distribution of current for a wide range of lengths and ratios of length to radius

* Decimal classification: R090. Original manuscript received by the Institute, January 10, 1944. This report is based on material from the 1943 Annual Review Committee of the Institute of Radio Engineers, as co-ordinated and edited by Laurens E. Whittemore and Keith Henney.

were given. These included curves showing the components of current in phase with the driving-potential difference and in quadrature with this, and curves giving the magnitude of the current and its phase angle referred to the driving-potential difference. The conventionally assumed sinusoidal distribution of current was shown to be a fair approximation for extremely thin antennas and for thicker antennas which do not greatly exceed $\lambda/2$ in length.

(6) R. King and C. W. Harrison, Jr., "The distribution of a current along a symmetrical center-driven antenna," PROC. I.R.E., vol. 31, pp. 548–567; October, 1943.

The analysis previously made for the current distribution along a symmetrical center-driven antenna of nonvanishing radius, and radiation field thereof, was extended to include long-wire center-driven antennas. The results of this investigation were then applied to obtain an approximate solution for the field of a long-wire resonant vee antenna.

(7) C. W. Harrison, Jr., "The radiation field of long wires, with application to vee antennas," *Jour. Appl. Phys.*, vol. 14, pp. 537–544; October, 1943.

The analytically difficult problem of determining the distribution of current in and the impedance of cylindrical antennas was further illuminated by a series of papers.

(8) N. Wells, "Discussion on 'aerial characteristics'," *J.I.E.E.*, (London), vol. 90, pp. 24–25; March, 1943.
(9) L. Brillouin, "The antenna problem," *Quart. Appl. Math.*, vol. 1, pp. 201–214; October, 1943.
(10) E. T. Glas, "On radiation problems concerning vertical antennas," (dissertation), published by Royal Administration of Swedish Telegraphs, Stockholm, 1943.

International Broadcasting

In the United States all international broadcasting continued to be supervised and controlled by the Office of War Information and the Office of the Co-ordinator of Inter-American Affairs. The number of transmitting stations, the facilities at each, and in several cases the operating powers were substantially increased. With few exceptions the antennas are of the rhombic type because of its flexibility, simplicity, and economy in critical materials.

Frequency-Modulation Broadcasting

Frequency-modulation broadcast stations licensed in the past in the United States continue to operate, although in some cases operating schedules were curtailed. Applications to the Federal Communications Commission for postwar construction permits accumulated, some being original applications but many being applications for reinstatement of permits which had been canceled since 1941.

Frequency-Modulation Transmitters and Receivers for Emergency Service

An interesting application of radio for inshore navigation was reported. United States Coast Guard buoys anchored near channel and harbor entrances were equipped with dual 5-watt transmitters operating between 286 and 315 kilocycles. These buoy radio beacons aided ships with radio direction finders through difficult waters when other means failed. The transmitters were powered with 14-volt storage batteries of sufficient capacity for 3 or 4 months of continuous operation. The two transmitters utilized a common output tank circuit and were alternately operated at about 7-second intervals to flash identification signals consisting of 1000-cycle modulation tone. Failure of either transmitter was evidenced by a longer interval between signals from the buoy.

(11) "Buoy radiobeacons for inshore navigation," *Electronics*, vol. 16, pp. 88–91; July, 1943.

TABLE I

RADIO BROADCAST STATIONS FOR WHICH LICENSES AND CONSTRUCTION PERMITS ISSUED BY THE FEDERAL COMMUNICATIONS COMMISSION WERE OUTSTANDING ON DECEMBER 31, 1943

Class of Broadcast Station	Number of Licenses	Number of Construction Permits
Standard	910	2
Commercial high-frequency (Frequency-modulation)	42	8
Experimental high-frequency (Including 1 station operating under "special authorization" and 5 stations operating under "temporary class 2" licenses)	8	0
Commercial television	5	3
Experimental television	21	6
International	18	11
Facsimile	3	0
Noncommercial educational	6	1

Frequency Modulation

Several papers of interest in the field of frequency modulation appeared during the years 1942–1943. Their emphasis was on the theory and operation of frequency modulation, and on specific pieces of frequency-modulation equipment in commercial use.

Frequency-modulated transmitters were improved by means of the Crosby push-pull circuit, such as is used in Philadelphia's frequency-modulation station, W69PH. This development, by using a push-pull circuit in conjunction with a reactance tube, balances out carrier-frequency instability due to power-supply variations. The high degree of performance attained by this circuit recommends it for extension in new transmitters. An interesting detail of this particular transmitter installation is the method of tuning. Plate-tank tuning is accomplished by varying the capacitance between the metal shields in which the tube anodes are mounted.

(12) M. G. Crosby, "Reactance-tube frequency modulators," *RCA Rev.*, vol. 5, pp. 89–96; July, 1940.
(13) E. S. Winlund and C. S. Perry, "A modern 10-kw frequency-modulation transmitter," *Electronics*, vol. 15, pp. 40–43; March, 1942.

Studio-to-transmitter radio circuits were improved by means of high-fidelity transmission of studio programs to the main transmitters.

(14) "F-M in S-T relay systems," *Communications*, vol. 22, pp. 16 and 18; July, 1942.
(15) W. F. Goetter, "Frequency-modulation transmitter-receiver for studio-to-transmitter relay system," PROC. I.R.E., vol. 31, pp. 600–607; November, 1943.

The problems encountered in the development of one of the latest station monitors as well as a discussion of its underlying theory were reported. The unit described measured the mean frequency of the frequency-modulated wave, always a difficult thing to do, by averaging over a period long compared with the period of the lowest modulating frequency.

(16) H. R. Summerhayes, Jr., "260- to 350-megacycle converter unit for General Electric frequency-modulation station monitor," PROC. I.R.E., vol. 31, pp. 249–253; June, 1943.
(17) H. R. Summerhayes, Jr., "A frequency-modulation station monitor," PROC. I.R.E., vol. 30, pp. 399–404; September, 1942.

By using a high degree of negative feedback in the intermediate-frequency section, it was shown possible to minimize the intermediate-frequency bandwidth, as well as to make the detected output independent of amplitude without the use of a limiter stage. The method proposed was an extension of the same idea proposed in 1939 by Carson and Chaffee.

(18) D. A. Bell, "Reduction of band width in F.M. receivers," *Wireless Eng.*, vol. 19, pp. 497–502; November, 1942.
(19) J. R. Carson, "Frequency-modulation: theory of the feedback receiving circuit," *Bell Sys. Tech. Jour.*, vol. 18, pp. 395–403; July, 1939.
(20) J. G. Chaffee, "The application of negative feedback to frequency-modulation systems," *Bell Sys. Tech. Jour.*, vol. 18, pp. 404–437; July, 1939.

Other published papers contained reviews of the theory and application of this type of transmission. These papers were concerned with a general discussion of amplitude, frequency, and phase modulation, with particular emphasis on the modes of operation, modulation factor, and the general frequency spectrum. Several reactance-tube networks were discussed, with emphasis being placed on the physical operation of these tubes as applied to modulated oscillators and amplifiers. Interference suppression in both amplitude-modulated and frequency-modulated systems was analyzed.

(21) S. W. Seeley, "Frequency modulation," *RCA Rev.*, vol. 5, pp. 468–480; April, 1941.
(22) A. Hund, "Amplitude, frequency, and phase modulation relations," *Electronics*, vol. 15, pp. 48–54; September, 1942.
(23) E. C. Cherry and R. S. Rivlin, "Non-linear distortion with particular reference to the theory of frequency modulated waves," part I, *Phil. Mag.*, vol. 32, pp. 265–281; October, 1941; part II, vol. 33, pp. 272–293; April, 1942.
(24) J. D. Weston, "Response of reactive networks to frequency-modulated signals," *Wireless Eng.*, vol. 19, pp. 251–253; June, 1942.
(25) H. Salinger, "Transients in frequency modulation," PROC. I.R.E., vol. 30, pp. 378–383; August, 1942.
(26) A. Hund, "Reactance tubes in F-M applications," *Electronics*, vol. 15, pp. 68–71, 143; October, 1942.
(27) H. J. Reich, "Interference suppression in A-M and F-M," *Communications*, vol. 22, pp. 7, 16, 19, 20; August, 1942.

Among other papers which relate to various applications of frequency modulation, the following may be noted.

(28) R. J. Pieracci, "A stabilized frequency-modulation system," PROC. I.R.E., vol. 30, pp. 76–80; February, 1942.
(29) A. A. Skene and N. C. Olmstead, "A new frequency-modulation, broadcasting transmitter," PROC. I.R.E., vol. 30, pp. 330–335; July, 1942.
(30) R. F. Guy and R. M. Morris, "NBC frequency-modulation field test," *RCA Rev.*, vol. 5, pp. 190–226; October, 1940.
(31) D Phillips, "Grounded-plate F-M amplifier notes," *Communications*, vol. 22, pp. 49–50; October, 1942.
(32) H. DuVal, Jr., "The tests that proved F-M vital to communications," *Communications*, vol. 22, pp. 5–7, 30; February, 1942.

Electronics

Cathode-Ray Tubes and Television Tubes

During the year 1943 the major effort in the development of cathode-ray tubes was expended on the group of preferred types for the armed services. Practically all of the preferred types were put into production and most of them were being made in rather large volume. The Cathode-Ray Tube Committee of the Radio Manufacturers Association in the United States was quite active in standardizing new test methods and in making recommendations on specifications.

The magnetic-deflection and focus types of tubes became very popular and demonstrated the advantages of their improved operating characteristics. Several of the electrostatic-deflection and focus types have shown the advantages of the special features which were introduced into their design, namely, better insulation, higher voltage operation, higher light output, better focus, higher deflection sensitivity, and improved high-frequency operation.

Two small desk-type electron microscopes were described and a number of papers appeared showing the varied applications of this new research tool.

(33) V. K. Zworykin, "Electron microscopy in chemistry," *Electronics*, vol. 16, pp. 65–68, 190–196; January, 1943.
(34) C. H. Bachman and S. Ramo, "Electrostatic electron microscopy. II," *Jour. Appl. Phys.*, vol. 14, pp. 69–77; February, 1943.
(35) C. H. Bachman and S. Ramo, "Electrostatic electron microscopy. III," *Jour. Appl. Phys.*, vol. 14, pp. 155–160; April, 1943.
(36) R. G. Picard and O. S. Duffendack, "Studies on the structure of thin metallic films by means of the electron microscope," *Jour. Appl. Phys.*, vol. 14, pp. 291–305; June, 1943.
(37) H. Green and E. F. Fullam, "Some applications of the high resolving power of the electron microscope," *Jour. Appl. Phys.*, vol. 14, pp. 332–340; July, 1943.
(38) C. Marton and S. Sass, "A bibliography of electron microscopy," *Jour. Appl. Phys.*, vol. 14, pp. 522–531; October, 1943.

Large High-Vacuum Tubes

In the field of large high-vacuum tubes, publication of developments, especially in tubes for ultra-high frequencies, was curtailed by wartime restrictions.

Because of the increasing use of high-frequency power in industry, the trend was toward larger, sealed-off tubes. Tubes of 500 kilowatts and more were being developed with emphasis toward longer life.

Ultra-high-frequency power tubes continued to be an active field of development, the tendencies being toward higher peak emission and higher powers.

(39) H. E. Hollmann, "Erzeugung und verstarkung von dezimeter und zentimeterwellen," *Telegraphen-Fernsprech—Funk und Fernseh. Technik*, vol. 2, pp. 281–293; November, 1942; vol. 2, pp. 322–331; December, 1942.
(40) M. Johnson, "Physical foundations of radio. III. Hot cathode emission (metallic)," *Wireless World*, vol. 48, pp. 259–261; November, 1942.
(41) M. Johnson, "Physical foundations of radio. IV. Composite cathodes, photosensitive surfaces, cold emission," *Wireless World*, vol. 48, pp. 291–293; December, 1942.
(42) G. Bocking, "Velocity-modulation U-H-F generators," *Wireless World*, vol. 49, pp. 16–18; January, 1943.
(43) H. J. Nolte, "Glass strain," *Gen. Elec. Rev.*, vol. 46, pp. 275–280; May, 1943.
(44) G. Chevigny, "Tubes for high-power short-wave broadcast stations—their characteristics and use," PROC. I.R.E., vol. 31, pp. 331–340; July, 1943.

(45) O. H. Schade, "Analysis of rectifier operation," PROC. I.R.E., vol. 31, pp. 341–361; July, 1943.
(46) R. I. Sarbacher and W. A. Edson, "Tubes employing velocity modulation," PROC. I.R.E., vol. 31, pp. 439–452; August, 1943.
(47) R. I. Sarbacher, "Power-tube performance in class C amplifiers and frequency multipliers as influenced by harmonic voltage," PROC. I.R.E., vol. 31, pp. 607–625; November, 1943.
(48) W. C. White, "Electron tubes," *Radio News*, vol. 30, pp. 50–51, 102; November, 1943.

Gas-Filled Tubes

The majority of the published work on gas-filled types related to tube circuits and methods of use rather than to fundamental theories of discharge in gases.

One new application brought forward in 1943 was the use of the Thyratron for speed control of direct-current motors. Two papers relating to this application were:

(49) E. E. Mayer, "Electronic control of d-c motors," *Electronics*, vol. 16, pp. 98–103, 215–217; May, 1943; pp. 119–125; June, 1943; pp. 118–122; July, 1943; pp. 133–138, 281–283; September 1943; pp. 128–133, 312–313, 316; October, 1943.
(50) E. H. Vedder and K. P. Puchlowski, "Rectifier analysis—for direct current motor drive," *Radionics*, vol. 1, pp. 3–7, *Radio News*, vol. 30, August, 1943.

Papers relating to power installations of tank-type rectifiers were:

(51) R. D. Evans, "Harmonics and load balance of multiphase rectifiers," *Trans. A.I.E.E.* (*Elec. Eng.*, April, 1943), vol. 62, pp. 182–187; April, 1943.
(52) A. Gaudenzi, "High-vacuum mutators for direct-current transmission," *Brown Boveri Rev.*, vol. 28, pp. 319–322; October, 1941.
(53) H. Keller, "The high-power mutator for direct-current power transmission," *Brown Boveri Rev.*, vol. 28, pp. 322–325; October, 1941.

A paper regarding stored-energy-type electronic-welding control was:

(54) J. W. Dawson and H. Klemperer, "Variable waveform unit for testing aluminum welding," *Electronics*, vol. 16, pp. 62–66, 201; February, 1943.

Papers which assisted in the understanding of gas-filled tube phenomena were:

(55) J. D. Cobine and E. C. Easton, "Time lag of impulse breakdown at high pressures," *Jour. Appl. Phys.*, vol. 14, pp. 321–331; July, 1943.
(56) E. Gerecke, "Mercury steel-tank mutators filled with rare gases," *Bull. Ass. Suisse Elect.*, vol. 33, pp. 226–228, discussion, pp. 228–230; April 22, 1942.
(57) H. Y. Fan, "Thermionic emission from an oxide-coated cathode," *Jour. Appl. Phys.*, vol. 14, pp. 552–560; October, 1943.

Photoelectric Devices

There was an expanding use of photoelectric devices in industrial applications.

A contribution to the understanding of the cesium-antimony photoelectric surface appeared in which a maximum sensitivity was reported for the stoichiometric ratio of 3:1 according to the relationship $Sb\ Cs_3$.

(58) A. Sommer, "Photo-electric alloys of alkali metals," *Proc. Phys. Soc.*, vol. 55, pp. 145–154; March, 1943.

A number of papers appeared in which the photoelectron multiplier was employed as a measuring device in scientific apparatus such as spectroscopic equipment.

Small High-Vacuum Tubes

In the field of small high-vacuum tubes, the most important feature of 1943 was probably the influence of the war on tube design, manufacturing, testing, and correlated matters essential to obtaining the best product for use in fighting a war of global proportions. In tube design and manufacturing, especial attention was given to tubes of sturdier construction, smaller size, improved ultra-high-frequency performance, reduced noise, and greater uniformity between the products of different manufacturers. Substantial progress was made in eliminating special tube selections required to service some military equipment. Much attention was given to the design requirements of tubes expected to perform well under all combinations of high and low temperatures, low barometric pressure, high humidity, extreme shock and vibration, and the attack of insects, fungi, and salt spray.

For new tubes which can be mentioned, conventional designs were employed but the trends of previous years toward closer spacings between electrodes, more precise construction, and reduction in size continued. It is of interest to note that the space requirements for conventional heater-cathode types of radio-frequency amplifier tubes have in the period from 1929 to date dropped from approximately 16.5 cubic inches to about 1.2 cubic inches, a reduction of over 90 per cent and that the heater power requirements have decreased in the same period from approximately 4.5 watts to about 1 watt, a reduction of over 75 per cent. There was a large increase in production of miniature tubes, i.e., tubes $\frac{3}{4}$ of an inch in diameter with 7-pin button seal for the leads, without an external base.

Television

A curtailed schedule of broadcast television programs was maintained in the United States in 1943. The principal activity took place in the New York area, but there was also broadcasting in Schenectady, Philadelphia, Los Angeles, and Chicago. Programs originating in New York were again regularly relayed and rebroadcast at Schenectady and Philadelphia.

Broadcasting equipment was essentially unchanged, except that some remote-pickup camera equipment, designed prior to the end of 1941, was first used on the air during 1943.

In the New York area, stations WNBT (50 to 56 megacyles), WCBW (60 to 66 megacycles), and W2XWV (78 to 84 megacycles) have provided a weekly period of simultaneous transmission of their test patterns. These simultaneous transmissions permit receiver installation and servicemen to obtain optimum performance on all these channels on one call.

Approximately 145,000 "Air Wardens" have received a portion of their training through a series of lectures and films broadcast via television in co-operation with the New York City Police Department. A comparable number of "Fire Guards" are expected to be reached by similar means.

Television receivers have been placed in six Army or Navy hospitals in the New York area to provide additional entertainment for convalescent servicemen.

A type of cathode-ray control of television light valves was described early in the year. This tube provided a means for the control of a high-intensity light source for the projection of large-size television pictures of high brightness.

The spectral characteristics of cathode-ray-tube screens for color television and their requirements for good color reproduction were published.

Field tests carried on during the year have indicated that the lower-frequency television channels are more suitable for high-quality reception, principally because of greater freedom from multipath reception.

(59) C. L. Townsend, "Contemporary problems in television sound," PROC. I.R.E., vol. 31, pp. 3–7; January, 1943.
(60) G. L. Beers, "Focusing view-finder problem in television cameras," PROC. I.R.E., vol. 31, pp. 100–106; March, 1943.
(61) H. A. Breeding, "Mercury lighting for television studios," PROC. I.R.E., vol. 31, pp. 106–112; March, 1943.
(62) O. H. Schade, "Radio-frequency-operated high-voltage supplies for cathode-ray tubes," PROC. I.R.E., vol. 31, pp. 158–163; April, 1943.
(63) J. S. Donal, Jr., "Cathode-ray control of television light valves," PROC. I.R.E., vol. 31, pp. 195–208; May, 1943.
(64) J. S. Donal and D. B. Langmuir, "A type of light valve for television reproduction," PROC. I.R.E., vol. 31, pp. 208–214; May, 1943.
(65) P.C. Goldmark, E. R. Piore, J. M. Hollywood, T. H. Chambers, and J. J. Reeves, "Color television—Part II," PROC. I.R.E., vol. 31, pp. 465–478; September, 1943.
(66) R. E. Shelby, "Television; far seeing eye of the future," *Electronics*, vol. 16, pp. 96–99, 178; March, 1943.
(67) R. L. Smith, "Three years of television relaying," *Electronics*, vol. 16, pp. 122–125, 277; September, 1943.
(68) N. Wells, "Short-wave dipole aerials," *Wireless Eng.*, vol. 20, pp. 219–232; May, 1943.
(69) P. Nagy and M. J. Goddard, "The signal converter. New thermionic discharge tube for the production of the time base deflection potentials of a cathode-ray tube," *Wireless Eng.*, vol. 20, pp. 273–299; June, 1943.

Facsimile

Commercial radiophoto and facsimile-radio-circuit facilities between United States and foreign countries were increased during 1942 and 1943. The standards of service were improved to a point not theretofore attained. Additional circuits connecting New York with Switzerland, Sweden, Cairo, and Brazil were put in operation. In the Pacific area, radiophoto circuits were placed in operation connecting San Francisco with China and Australia. A commercial radiophoto circuit between Honolulu and San Francisco was also in operation. Radiophoto facilities of the United States Army were used to augment commercial circuits in the delivery of public-interest news pictures from the Mediterranean and Southwest Pacific areas to the United States. The United States Office of War Information commenced an extensive radiophoto-broadcast program to various parts of the world.

The wire-photo services operated by the newspaper-publishing companies were continued. Quality and speed of service were maintained despite loss of personnel to the Military Services.

During the past year more than one million telegrams were handled by telefacsimile installations in operation in New York, Chicago, Atlanta, and San Francisco. Telefacsimile equipment was also performing a useful service in the train-dispatching field. In this application the facsimile transmitter, operated by the railroad telegraph operator in a long section or division, was arranged to transmit train orders to any one of a number of recorders, all operating on a single-line pair and located at strategic points along the right of way. The operator would select the desired recorder by dialing, set the transmitter for the desired number of copies (one for each member of the train crew), and deposit the order into the facsimile transmitter.

Temporary Facsimile Test Standards of the Institute were formulated for publication.

Piezoelectricity

A survey was made of the minerals found in the United States, with special reference to their piezoelectric properties. Out of 830 minerals tested, only 17 showed definite piezoelectric properties. Of these, 14 were reported for the first time. None of them held out promise of useful applications.

Considerable progress was made toward a standard terminology with respect to the axes and other properties of piezoelectric crystals, and their experimental determination.

The applications of quartz crystals received considerable attention, both in the form of survey articles and in investigations of a more specialized nature. The use of X rays for determining the orientation of quartz crystals; and the inspection of quartz crystals to determine flaws, veils, and optical and electrical turning were discussed.

On the theoretical side, a study was made of the mathematical treatment of the physical properties of crystals in general. Further progress was also made in the theoretical treatment of vibrating quartz plates, taking into account the interconnection between the various vibrational modes. In the main, the theoretical conclusions were found to be in good agreement with experimental results.

Among recent applications of Rochelle salt may be mentioned a recording oscillograph, intended especially for use with a surface analyzer. To reduce or neutralize electrical leakage of Rochelle-salt crystals under service conditions, methods have been developed for wrapping the crystal elements in metal foil insulated from one or both electrodes, and also for impressing a counter-potential on a guard electrode.

(70) C. K. Gravley, "An instrument for measuring surface roughness," *Electronics*, vol. 15, pp. 70–73; November, 1942.
(71) J. H. Ream, "Piezoelectric unit," U. S. Patent No. 2,324,024, July, 13, 1943.
(72) J. P. Arndt, "Leakage reducing means," U. S. Patent No. 2,289,954, July 14, 1942.
(73) W. L. Bond and E. J. Armstrong, "Use of X-rays for determining the orientation of quartz crystals," *Bell Sys. Tech. Jour.*, vol. 22, pp. 293–337; October, 1943.
(74) G. W. Willard, "Raw quartz, its imperfections and inspection," *Bell Sys. Tech. Jour.*, vol. 22, pp. 338–361; October, 1943.
(75) W. L. Bond, "A mineral survey for piezo-electric materials," *Bell Sys. Tech. Jour.*, vol. 22, pp. 145–152; July, 1943.
(76) W. G. Cady and K. S. Van Dyke, "Proposed standard conventions for expressing the elastic and piezoelectric properties of right and left quartz," PROC. I.R.E., vol. 30, pp. 495–499; November, 1942.

(77) W. L. Bond, "Methods for specifying quartz crystal orientation and their determination by optical means," *Bell Sys. Tech. Jour.*, vol. 22, pp. 224–262; July, 1943.
(78) E. W. Kammer and J. V. Atanasoff, "A determination of the elastic constants of beta-quartz," *Phys. Rev.*, vol. 62, pp. 395–400; October 1 and 15, 1942.
(79) W. P. Mason, "Quartz crystal applications," *Bell Sys. Tech. Jour.*, vol. 22, pp. 178–223; July, 1943.
(80) J. E. Benson, "Modes of vibration and design of V-cut quartz plates for medium broadcast frequencies," *A.W.A. Tech. Rev.*, vol. 6, no. 2, pp. 73–90; 1943.
(81) F. E. Fox and G. D. Rock, "A quartz plate with coupled liquid column as a variable resonator," PROC. I.R.E., vol. 30, pp. 29–33; January, 1942.
(82) R. Bechmann, "Properties of quartz oscillators and resonators in the range from 300 to 5000 kc," *Hochfrequenz. und Elektroakustik*, vol. 59, pp. 97–105; April, 1942.
(83) R. Bechmann, "Longitudinal vibrations of square quartz plates," *Zeit. für Phys.*, vol. 118, pp. 515–538; February 1, 1942.
(84) R. Bechmann, "Longitudinal vibrations of rectangular quartz plates," *Zeit. für Phys.*, vol. 120, pp. 107–120; November 16, 1942.
(85) W. L. Bond, "The mathematics of the physical properties of crystals," *Bell Sys. Tech. Jour.*, vol. 22, pp. 1–72; January, 1943.

Acknowledgments

This summary of progress during 1943 covers, in general, the period up to the first of November and is necessarily limited chiefly to certain developments in the United States which can appropriately be published under present circumstances. It is based on material prepared by members of the 1943 Annual Review Committee of the Institute of Radio Engineers, edited and co-ordinated by Laurens E. Whittemore and Keith Henney.

The other members of the Annual Review Committee for 1943 and the committees of the Institute of which they are chairmen are as follows:

R. S. Burnap	Electronics
C. R. Burrows	Radio Wave Propagation
W. G. Cady	Piezoelectric Crystals
J. L. Callahan	Facsimile
C. C. Chambers	Frequency-Modulation
C. J. Franks	Radio Receivers
R. F. Guy	Transmitters and Antennas
I. J. Kaar	Television
G. G. Muller	Electroacoustics
H. M. Turner	Symbols
H. A. Wheeler	Standards

A. F. Van Dyck

The chairmen of the above committees wish to acknowledge the assistance given them by individual members of the committees.

Spectrographic Analysis in the Manufacture of Radio Tubes*

S. L. PARSONS†, NONMEMBER, I.R.E.

Summary—**This paper describes the use of spectrographic methods in attacking some of the problems encountered in the manufacture of radio tubes. Illustrations of the laboratory and apparatus are discussed. The techniques and use of qualitative spectrographic analysis in connection with chemical, metallurgical, ceramic, and fluorescent problems are described and illustrations of the application of quantitative spectrographic analysis to problems of routine inspection and control are given.**

THE branch of optics known as spectroscopy has had a long and illustrious career since Newton, in 1666, produced the first spectrum. Not only has the spectrograph been used in discovering several new elements, notably cesium, rubidium, and showing helium in the sun before it was isolated on earth, but it is well known as having provided most of the evidence used in formulating our present theories of atomic structure. Spectrographic analysis had its beginning when it was found that the spectrum of the light emitted by an atom was as characteristic of that atom as a "fingerprint." The identification of these spectral fingerprints then gave a means of qualitative chemical analysis while the discovery that the intensity of the spectral lines is a function of the amount of element present, provided a means of quantitative analysis. Early applications of spectrographic techniques to industrial problems were made without too careful attention to the limitations of the method. This caused it to fall into bad repute so that it was only used for qualitative analysis, and rough quantitative analysis where the concentrations were low and high precision was not necessary. Research carried forward during the last ten years has shown that the spectrograph can be used with considerable advantage in many industrial problems.

The spectrographic equipment was installed in the engineering research laboratory of Sylvania Electric Products with the thought in mind that it would be used as a tool in attacking chemical, metallurgical, fluorescent, and ceramic problems arising in the manufacture of radio tubes and lamps.

It was soon found that the laboratory would have to be equipped to perform two functions. First, since a large number of jobs are handled that are of a trouble-shooting nature, rather flexible equipment is needed to investigate the various types of problems. This means making qualitative analyses and developing methods and procedures for routine quantitative analysis, a definite research setup. Second, once the proper methods have been worked out, equipment must be available to perform analyses of a routine nature as a control of the

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materials used in manufacturing radio tubes. The spectroscopic section also has a long-range program involving the application of the techniques of absorption spectroscopy. However, at the present time we are only discussing the use of emission spectroscopy.

Before going into the discussion of the use made of the spectrograph in attacking manufacturing problems, it may be of interest to see some of the equipment as set up in our laboratory for this work. Fig. 1 shows the spectrograph and arc bench in use at present. The spectrograph is a Bausch and Lomb instrument of the Littrow type equipped with both glass and quartz optics so that the region from 2000 to 10,000 angstroms can be covered with adequate dispersion for almost any type of material. The electrode stand is arranged on ways so that the distance from the arc to the spectrograph slit can be easily varied. This makes a simple, reproducible means of controlling the exposure so that representative sampling of the material in the arc is obtained. All other source equipment, such as a hydrogen discharge lamp,

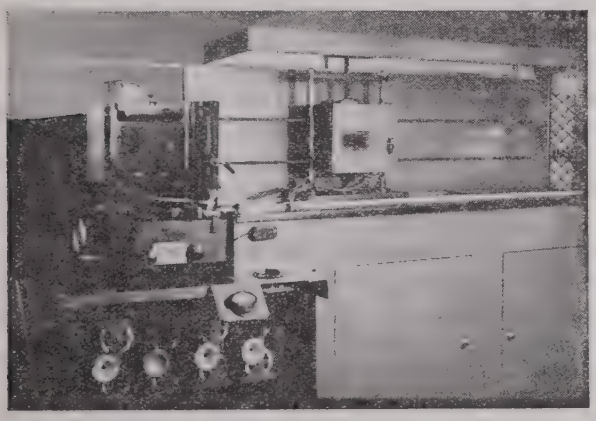

Fig. 1—Spectrograph and arc bench.

rotating sectors, etc., are arranged with bases that fit on the ways so that the desired apparatus can be set in position on the arc bench. A hood is provided over the arc bench to carry away all fumes and vaporized material. This has proved to be very effective in preventing contamination of a sample due to previous analyses. The electrical equipment used to provide the various types of sources is isolated in the region behind the spectrograph and arc bench. Leads carrying the high voltage come through slots in the panel behind the arc stand and since all doors leading to the source area are equipped with safety switches which break the circuit if opened. the operators are protected from accidental contact with the high voltage. The types of sources provided are a 250-volt direct-current motor generator capable of supplying 50 amperes, which is used mainly for qualitative analysis, a high-voltage transformer supplying 1100, 2200, and 4400 volts alternating current at currents from 1.5 to 6 amperes used for quantitative analysis, and several small power supplies used for exciting Geisler tubes, a hydrogen-discharge lamp, ultraviolet mercury arcs, and other gaseous discharges. There is

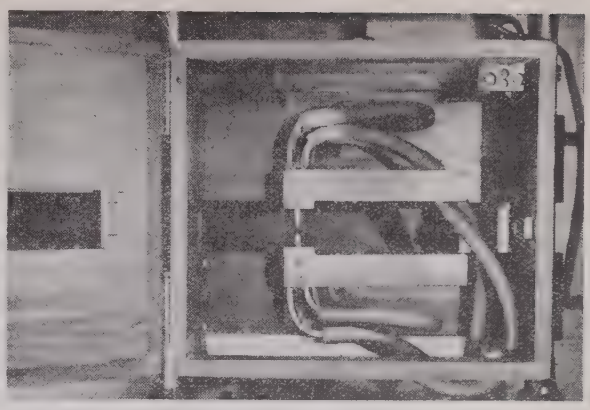

Fig. 2—Electrode holders.

now being installed a 40,000-volt interrupted-spark source after the design of Vincent and Sawyer,[1] which is not yet completed. The controls for these sources are mounted on a panel at the far end of the arc bench where they are readily accessible. The doors shown under the arc bench give access to various units that are at high potential and are also equipped with safety switches.

Fig. 2 shows a close-up of the arc stand which holds the electrodes in position to be arced. A pair of carbon electrodes is shown clamped in the electrode holders. This pair of holders is equipped with water-cooling coils which lower the temperature of the electrodes and help to reduce background on the spectrographic plate. The electrode holders are readily removable and another set without water cooling can be installed when the high-voltage spark is used. The lower holder is fixed, while the upper one is so mounted that it can be raised and lowered by means of a screw. This makes it possible to control the spacing of the carbon electrodes accurately. The whole assembly is enclosed in a protective shield, the door to which is shown swung open on the left. This door is also equipped with a safety switch which shuts off the arc supply when opened. The housing not only protects the operator from accidental contact with the high potential but also shields anyone working in the

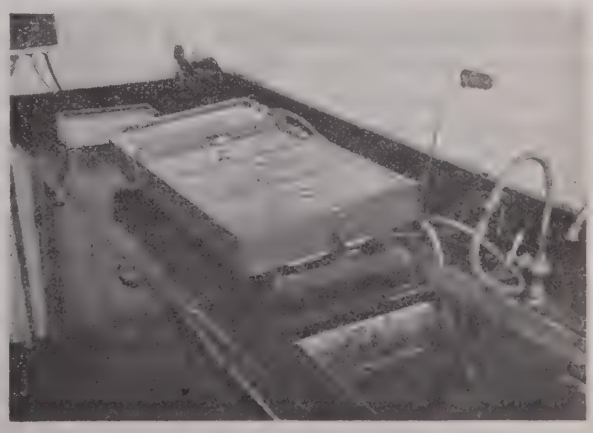

Fig. 3—Photographic developing equipment.

[1] H. B. Vincent and R. A. Sawyer, "The spectrograph in the iron foundry," *Jour. Appl. Phys.*, vol. 8, pp. 163–173; March, 1937.

laboratory from the direct rays from the arc. The door to this shield has a small glass filter, of the type used in welders' helmets, inserted in it so that one may study the burning characteristics of the arc.

Fig. 3 shows a portion of the darkroom. Use is made of three developing solutions, developer, hardener, and fixing bath. The trays for these solutions are arranged to be automatically rocked by a motor to insure uniform agitation. The developer tray is made of stainless steel so that cooling water may be circulated in close contact with the developer in order to have a control over its temperature. With this setup, a very uniform plate processing has been secured, the calibration curve for one plate being almost identical to the next. The plate washer is shown at the back of the sink. It consists of a metal tube having a series of fine holes drilled in a line along one side. This produces a thin sheet of water across the plate and is quite effective in removing the fixer. Plates washed for two minutes in this device have kept

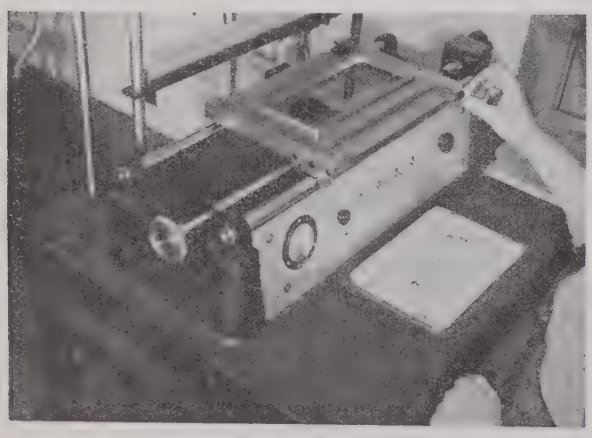

Fig. 4—Densitometer.

well over a period of three years. After washing, the plate is dried on a high-speed drier shown at the far end of the darkroom bench. The drier is merely a small transite box having nichrome heater coils in the bottom. The plate is laid in a frame on top of the box and the air stream from a fan blows across the top. This device will dry a plate so that it may be worked on in about two minutes. It is necessary, of course, to use a hardening solution after developing in order to keep the emulsion from shrinking. The refrigerator used for storing the photographic plates is not shown in this picture; however, it has been found indispensable since it makes possible the storing of plates for a year or more without marked deterioration of their sensitivity.

The densitometer used in making quantitative spectrographic analyses is shown in Fig. 4. This instrument is of our own design and was built in the company shop. Essentially, it consists of a framework which holds the plate in a horizontal position so that it may be moved in two directions on sets of ways. A projection lamp located underneath the table is arranged to illuminate an area one inch square in the plane of the plate. A set of lenses and first-surface mirrors above the plate then project an enlarged image of the illuminated area onto the screen in front of the operator. This screen has a small slit in its center, behind which is located a barrier-layer photoelectric cell. The output from the photoelectric cell is fed to a wall-type galvanometer whose deflections are observed by means of a light beam and translucent scale. The scale is shown just above and to the rear of the plate stage. A system of clutches and controls is provided which makes it possible to shift rapidly to any desired portion of the plate. In operation, the plate is merely shifted about until the desired line appears at the edge of the photocell slit. At the turn of a switch, a motor drive causes the line to move across the slit at a uniform rate. As the line traverses the slit, the operator notes the maximum deflection of the galvanometer. This deflection is then used in calculating the concentration of the element in question. The instrument has a heavy cast-iron base and is very stable; it has required very little maintenance over the past two years.

One of the main problems in applying spectroscopy to industry is the question of its possibilities and limitations. Some seventy elements are detectable with an instrument having the dispersion and range of the one just described. It is rather embarrassing, however, to have to turn down requests for analyses of an organic impurity or some element such as sulfur or the halogens. The explanation that these elements can only be handled by rather specialized techniques is apt to raise a question as to the usefulness of the spectrograph. Fortunately, our chief chemist is familiar with spectroscopic techniques, and since all requests for analysis of a trouble-shooting nature come directly to him, he is able to refer those problems which we are equipped to handle directly to us while the chemical-analytical section handles those which we cannot do. The types of analysis which fall within the realm or the spectrograph have been thoroughly discussed with the heads of the metallurgical and ceramic sections so that they too are familiar with the service we can render. We are now called upon to apply spectrographic techniques to hundreds of different problems and have reached a point where the spectrographic laboratory is considered indispensable.

In discussing the application of the spectrograph, it probably will be best to divide the work done into two groups: those problems that only require qualitative analysis, and those that need quantitative analysis, or a combination of both.

Qualitative Analysis

Qualitative spectrographic analysis has proved itself very useful by providing clues which have led to solutions of various manufacturing troubles. It is characteristic of the method that it only requires a very small amount of sample and is capable of detecting almost infinitesimal amounts of about 70 of the 92 elements. A complete analysis covering all 70 elements could be made in a few hours. However, most samples only

require a check on about 25 elements so that sufficient information is obtained in less than an hour. A large number of qualitative analyses are made for the chemical laboratory which are used as pilot analyses for their quantitative determinations. These help them considerably in planning their analytical procedures since the presence or absence of certain elements calls for different techniques.

The factory frequently sends us radio tubes with small, mirror-like deposits on the insides of the bulbs, indicating the presence of some material evaporated from the parts during exhaust. These deposits are very small and require a sensitive method of analysis in order to determine what elements are present. By washing out the inside of the envelope with a small amount of dilute solution of nitric acid or aqua regia, and then evaporating the solution onto the flat ends of a pair of electrode carbons, it is possible to arc the electrodes and determine the elements present in the deposit. Such an analysis then makes it possible to track down the source of trouble, which may be contamination by copper from the welding electrodes or contaminations arising from the handling of plates or grids.

Qualitative analysis has proved itself extremely useful in connection with the base pins of lock-in radio tubes. Manufacturing requirements are at least one melt per month of the alloy from which the wire for these pins is drawn. Upon changing over to a new melt, it was found that the sealing qualities of the new wire were quite inferior to those of the wire used during the previous month. On running a comparison analysis of the two melts, the presence of a small amount of columbium was found in the inferior material. When the supplier checked back, it was found that $\frac{1}{2}$ per cent of columbium had inadvertently been added to aid in drawing the wire. Since then, all new melts of material have been carefully analyzed for columbium before they are put in production. Later an improved melt of material was tested which had superior sealing qualities to the ordinary run of pin alloy. Analysis of these pins showed an increase in the amount of aluminum present. This clue was followed by the metallurgical section to a point where it was proved that the optimum amount of aluminum is of the order of 2 per cent. A patent has been granted on this alloy.

The electron emission from oxide-coated cathodes frequently is not satisfactory due to the presence of impurities such as lead, iron, and silicon. The spectrograph is usually called upon to aid in the search for the place where these impurities are introduced. The hoppers through which the wire runs as it is coated with the oxides have been analyzed qualitatively for the presence of elements which could cause the contamination and it has been found necessary to reject a number of them. Of course, it has sometimes been possible to trace the presence of these contaminants back to the original material, and now at least a qualitative check analysis is run of each new stock of raw material as it is received by the company, while some lots are analyzed quantitatively.

The search for sources of contamination of the various materials inside a radio tube goes on unendingly. Samples come to us in a great variety of forms; there may be washings from plates, micas, or stems, which have been evaporated down so that only a small black spot remains in a beaker. There may be a section of filament support which has a metallic deposit on it, or cathodes which have strangely discolored areas. Since the spectrographic laboratory is somewhat separate from the other departments, its reports are unbiased and have helped solve many of these troubles.

Another use for qualitative spectral analysis is found in the ease and speed with which it can be used to identify various materials. Spectra of the many different alloys and glasses used in different types of tubes have been recorded and one merely has to burn the unknown material in the arc and then compare the photographed spectrum with those on file to identify the material. This procedure is also used in sorting mixed lots of material. An interesting case of using the spectrograph to detect mixed materials occurred recently. Four developmental tubes when life-tested showed rather surprising results. The stem of one tube cracked after about twenty hours' life, while the other three operated satisfactorily for over a thousand hours. Samples of the glass from the stems of all four tubes were qualitatively analyzed and the glass in the cracked stem was found to be of distinctly different composition. Upon checking back, it was found that one piece of stem tubing of an entirely different glass had been accidently included with the regular material. Subsequent tubes made with the correct glass proved entirely satisfactory.

Frequently, it is found that a certain type tube on the market differs in its characteristics from those which we manufacture, or perhaps we are requested to make tubes similar to those of foreign manufacture. When this occurs, the engineer whose job it is to design the tube usually obtains samples of the unknown type and dismembers them. Small samples of all the various materials within the tubes are then sent to us for qualitative check. This gives enough information to determine what alloys are used for the various parts.

Qualitative spectral analysis can be pushed to a point where it can be considered as semiquantitative. This has been done particularly in the analysis of materials used in making cathode-ray screens. In this case, samples are taken before and after a purifying procedure and are arced under identical conditions. It is then possible to compare the lines due to the various impurities and thus to determine whether or not the purification procedure is improving the material. Such a procedure has been used successfully for a period of over two years in work on the removal of iron, nickel, lead, manganese, and copper from zinc sulfate. At present, however, regular quantitative methods have been set up for these materials and will be discussed later.

Quantitative Analysis

Applications of quantitative spectrochemical analysis have not been of such a diversified nature as those discussed for qualitative analysis. The problems usually develop along the following lines. A qualitative study of the material over a period of time shows that some constituent or impurity in the material is of importance and it is them deemed necessary to perform periodic quantitative analysis on this component in order to be sure its concentration remains within specifications. A number of quantitative analysis procedures have been set up in our laboratory in co-operation with various other departments. In discussing the qualitative work, the importance of an optimum amount of aluminum in pin material was pointed out. A procedure by which lock-in pin alloy is analyzed quantitatively for aluminum has been set up. Techniques for the quantitative analysis of impurities present in the barium, strontium, and calcium carbonates used for emission coating on filaments, and a method of determining the concentration of thorium in tungsten wire have been developed. A number of other procedures are also in use. However, since they all embody approximately the same techniques, it is felt that by describing the method used in analyzing for the small amounts of copper present in cathode-ray-tube screens, this type of analysis can best be illustrated.

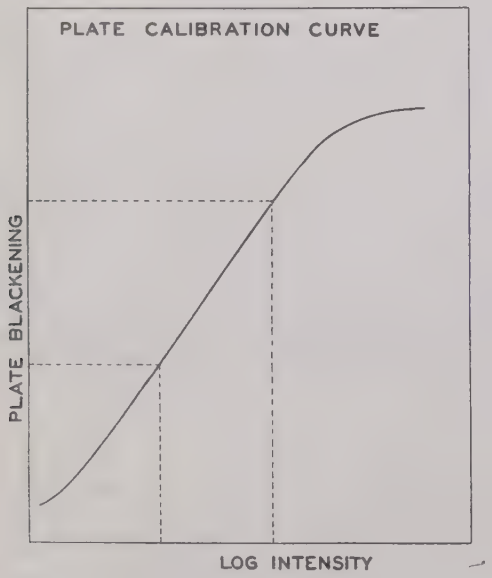

Fig. 5—Calibration curve.

In general, there are two methods of performing quantitative spectrographic analysis. In the first, one exposes several groups of standards on the same plate with samples of the unknown and then by visually comparing the intensities of the spectral lines due to the impurity, he is able to arrive at the concentration. The second method is known as the internal-standard method and involves an intensity measurement of a spectrum line due to the impurity and also of a line due to the major component of the sample. The ratio of these intensities is then obtained for a series of chemically analyzed or synthetically prepared standards and a curve is plotted relating intensity ratio and concentration. In analyzing an unknown, one only has to measure this intensity ratio and obtain the concentration from the curve. The second method is used in nearly all cases in our laboratory. The first method is used occasionally when it is wished to form an idea of the amount of some element present without spending too much time developing an analysis procedure.

In order to prepare cathode-ray-tube screens which will have the proper color in operation, it has been found necessary to have all the component materials of very high purity or at least of a controllable purity. The presence of small amounts of copper, iron, and nickel are sufficient to influence the color of the screen; therefore, considerable work has been done in arranging purification procedures for the raw material. The spectrograph is used in checking quantitatively the amounts of impurity present.

The amount of copper present in zinc sulfate runs around 0.00002 per cent or about two parts of copper to ten million parts of zinc sulfate. Since this is such a small amount, it has been found necessary to use the most sensitive source available. The only source which has proved suitable for this work has been the 250-volt direct-current arc using a 15-ampere arc current. In order to perform the analysis, the zinc sulfate is evaporated to dryness and packed into a crater drilled in the lower electrode, and a pointed, solid carbon is used as the upper electrode. Some difficulty was experienced in obtaining carbon which is free of copper. Electrode material was obtained from a large number of suppliers and experiments were also made to determine the suitability of chemical-purifying methods. It was finally found, however, that carbon electrodes as supplied by the Dow Chemical Company were suitable for this use. It is necessary to check each lot of carbons when it arrives as occasionally faint traces of copper are present which disturb the analysis. The sample is arced and photographed on an Eastman type 33 plate. In order to obtain the intensity ratio of the copper line at 3247 angstroms to the internal standard line, it is necessary to know exactly how the photographic plate responds to the intensity of the light incident upon it. This relationship must be known for each plate and is obtained in the following manner. A pair of spectroscopically pure iron electrodes having conically ground points are exposed on the plate using the 2200-volt alternating-current arc. The conditions have been chosen so as to be accurately reproducible. The relative intensities of ten lines in the iron spectrum of about the same wavelength as the copper impurity line have been determined by calibration against a step sector and a continuous source. To obtain the calibration curve for a plate, it is only necessary to read the blackenings of each of these lines and plot them against their assigned relative intensities. A typical calibration curve for the plates which are used is shown in Fig. 5.

In order to perform an analysis for copper, it is necessary to know how the copper impurity line varies with changes in concentration. This relationship is obtained by exposing a series of standards whose concentrations are known and measuring the intensity ratio for each standard. Plotting the concentration against the intensity ratio gives a curve of the form shown in Fig. 6. This curve shows the ratio of the intensity of the copper line 3247 to the intensity of the background. Theoretically, the best results would be obtained by taking the ratio of the copper line to a line due to the major component which in this case would be zinc. However, since the zinc spectrum has only a few lines in this region and those present are much too intense, it has been found that using the background as an internal standard gives much better results. The curve in Fig. 6 was prepared from a set of synthetically contaminated standards. The purest stock solution the fluorescent laboratory had prepared at the time of making the full-line calibration curve still contained a small amount of residual copper so that when known amounts of copper were added to this stock solution, the effect of the residual became marked in the lower concentrations. This is shown in the noticeable deviation from a straight line which occurs at the lower part of the curve. For the analysis of the various lots of material prepared in the plant, the continuation of the

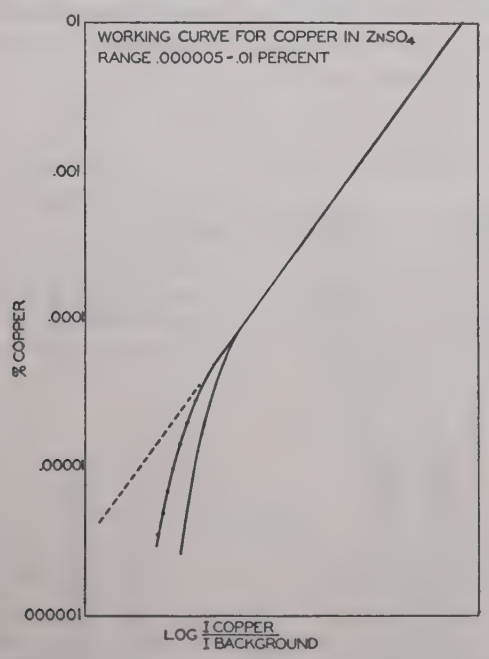

Fig. 6—Analytical curve.

straight line was considered as being the true value. It has been shown by other workers that a portion of this deviation from a straight line is due to the effect of background; however, in our work we consider the straight line to be of sufficient accuracy to be used in control analysis of the materials. After such analyses had been performed for a period of six months, the fluorescent laboratory had developed purification procedures to a point that their residual copper was reading about 0.00002 per cent instead of the 0.000035 per cent experienced formerly. A set of standards for the lower values was prepared from this solution and a new working curve was made. This curve is shown by the dotted line and is noticeably closer to the straight-line relationship we have assumed as correct.

In discussing quantitative analysis, the first question asked is "What is the accuracy?" Table I shows the results of twenty-four determinations of the same unknown sample which had an average concentration of copper of 0.000014 per cent. The standard deviation, figured on the basis of the twenty-four samples, is 7.8 per cent. This means that in a Gaussian distribution of random errors, 67 per cent of the results should deviate from the mean by not more than the standard deviation. The maximum error in this determination runs about ± 16 per cent. This accuracy has proved to be sufficient for checking the material used in cathode-ray screens. Some investigation has been started towards increasing this accuracy; however, at present we are more interested in keeping the sensitivity of our method adequate for the demand. Our biggest worry is that the fluorescent laboratory will achieve a purification procedure which will be so good that traces of copper present will not be shown.

TABLE I
SPECTROGRAPHIC DETERMINATION OF COPPER IN ZINC SULFATE

Number	Analysis	Deviation from Average	Per cent Deviation
1	0.0000140	0.0000005	3.4
2	0.0000160	0.0000015	10.3
3	0.0000150	0.0000005	3.4
4	0.0000152	0.0000007	4.8
5	0.0000172	0.0000027	18.6
6	0.0000130	0.0000015	10.3
7	0.0000148	0.0000003	2.7
8	0.0000140	0.0000005	3.4
9	0.0000145	0.0000000	—
10	0.0000152	0.0000007	4.8
11	0.0000150	0.0000005	3.4
12	0.0000148	0.0000003	2.7
13	0.0000122	0.0000023	15.8
14	0.0000140	0.0000005	3.4
15	0.0000150	0.0000005	3.4
16	0.0000130	0.0000015	10.3
17	0.0000140	0.0000005	3.4
18	0.0000138	0.0000007	4.8
19	0.0000156	0.0000011	7.6
20	0.0000142	0.0000003	2.7
21	0.0000148	0.0000003	2.7
22	0.0000128	0.0000017	11.7
23	0.0000130	0.0000015	10.3
24	0.0000156	0.0000011	7.6
	Average 0.0000145	Standard Deviation =7.8 per cent	5.9

Some of the analyses performed in this plant have been indicated. Since the materials are so varied, one must have available techniques and apparatus which are rather versatile in order to handle them. In most cases, an answer can be given within a couple of hours and on extremely important problems where speed is a major factor, definite information has been obtained in as short periods as half an hour. The spectrograph is of much use when it can be applied in connection with other branches of research. When the limitations and possibilities of the method are recognized throughout the organization, the application of the spectrographic technique can be made to pay large dividends.

Certain important radio-and-electronic developments are published in a form not readily available to or usable by many readers of the PROCEEDINGS. The Institute deems it desirable to publish at this time English translations of certain of the corresponding papers. Through the appreciated courtesy of the publishers of the original source, the *Journal of Technical Physics* (Russian), the following paper has been placed at the disposal of our readers. Thanks are also expressed to the translator of the paper, I. B. Bensen for his extensive and voluntary efforts.

The Editor

Generation of High-Power Oscillations with a Magnetron in the Centimeter Band*

N. F. ALEKSEEV†, NONMEMBER, I.R.E., AND D. D. MALAIROV†, NONMEMBER, I.R.E.
Translated by I. B. Bensen‡, NONMEMBER, I.R.E.

Summary—**Experiments are described for obtaining high-power oscillations from a four-cavity, water-cooled, demountable magnetron. Power outputs up to 300 watts at a wavelength of about 9 centimeters were measured. A sealed-off sample magnetron was built along similar lines and gave a power output of about 100 watts at the same wavelength under actual working conditions. An experimental magnetron was also built giving power output of two watts at a wavelength of about 2.6 centimeters.**

I. INTRODUCTION

THE WORK described in this article was done during the years 1936 and 1937 for the purpose of producing high-power oscillations in the centimeter-wave band by the use of magnetrons.

Previously published data indicated that 20 watts was the highest power obtainable at a wavelength shorter than 10 centimeters. The chief problem in obtaining higher power outputs has been the inability to obtain sufficient dissipation of power from the anode, a problem made more difficult by the low over-all efficiency. To increase the high-frequency output power, it was necessary to introduce water cooling of the anode and, if possible, increase the size of the anode without increasing the wavelength. In addition, it was necessary to choose the proper shape of the anode cavity. The magnetron tubes described in this article all incorporate the oscillating circuit within the tube.

The most promising modes of operation for a magnetron in the centimeter-wave band are of the so-called "electronic-oscillation" type or, as they are sometimes called, "oscillations of the first order." This is the case where $n=1$, the letter n designating the number of revolutions of the electron around its orbit during each complete electrical cycle. For this case of $n=1$, the theoretical relationship between wavelength and field strength is $H\lambda=10{,}700$ or $22{,}000$, depending upon the mode of oscillations in the anode structure of the magnetron.

This type of magnetron oscillation required relatively less powerful magnetic fields which was an important factor from the viewpoint of reducing the size of the magnets. In addition, it was found that powerful magnetic fields produce back heating of the cathode which constituted a limit to the output of this form of tube.

In the course of our experiments, oscillations for the case of $n=\frac{1}{2}$ were also discovered. In this case $H\lambda=6500$ or 9000. Therefore, one complete revolution of the electron around its orbit occurred during the time of two complete electrical cycles.

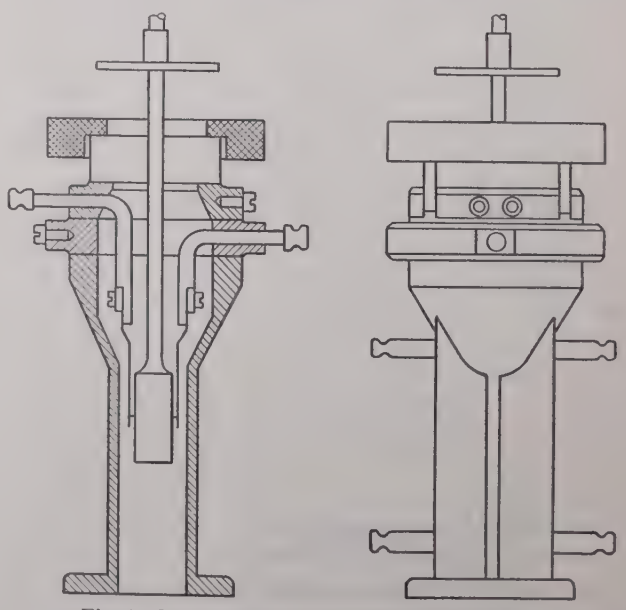

Fig. 1—Layout of the demountable magnetron.

Other workers in this field have also obtained such oscillations experimentally. For the very short wavelengths, this $n=\frac{1}{2}$ mode may well become more desirable than the oscillations of the first order ($n=1$).

In the case of magnetrons for the centimeter-wave band, it was impossible to determine analytically the dimensions of the anode elements with any degree of

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‡ Electronics Laboratory, General Electric Company, Schenectady, N. Y.

exactness so the optimum dimensions were determined experimentally.

II. Experimental Technique

In order to test results with different shape of electrodes, a demountable type of magnetron was built to operate with a continuously functioning vacuum pump. A cross section of this type of tube is shown in Fig. 1. The bottom flange of the magnetron was mounted directly on an oil-diffusion pump. The pumping system was operating continuously during the tests recorded here.

The anodes were made out of pure solid copper having circular cross sections. This latter feature allowed the greatest accuracy in manufacture. They were cooled by water flowing through a copper tube attached to the circumference of the cylindrical anode structure. The copper tubes used to supply this cooling water also served as electrical leads. The ends of these copper tubes extended outside the magnetron through a quartz disk (see Fig. 1).

An approximate determination of the power output of the tube was obtained from the brightness of incandescent load lamps. A more accurate determination was made from measurements of the increase in temperature of the cooling water. This latter method was made accurate by measuring both the input and output water temperature by two thermocouples. These thermocouples were connected differentially in the galvanometer circuit; therefore, they read directly the temperature rise in the water due to power dissipation in the tube. This method excluded possible errors which might have been introduced by fluctuations in the temperature of the incoming water.

By this method it was possible to determine very small water temperature differences and thus measure the power output even when the efficiency was as low as one or two per cent.

The measurements of wavelength were made by means of Lecher-wire systems.

III. Anode Shapes

The path of oscillations in the magnetron at these short wavelengths is determined by the design of the oscillating cavities in the anodes themselves. There are many possible variations in the shape of the anode cavities. In an anode with two slots, there may be one, two, or four actual resonant circuits. It is possible to connect in combination a large number of such cavities. An example of this is shown in Fig. 2 and such a design was checked experimentally although in this article this particular scheme is not described in detail as it is rather bulky although capable of producing very high powers.

High-power outputs were also obtained from what might be termed a single-cavity magnetron.

Regardless of the design of anode used, it is desirable to use a high anode potential to obtain high power outputs. This also allows an increase in the diameter of the cylindrical space in the anode material (d_k) for the accommodation of the cathode. This is because the anode potential E_b, the diameter of the hole for the cathode in the anode block, and the magnetic field strength H, are related by the equation $E_b = KH^2d_k^2$.

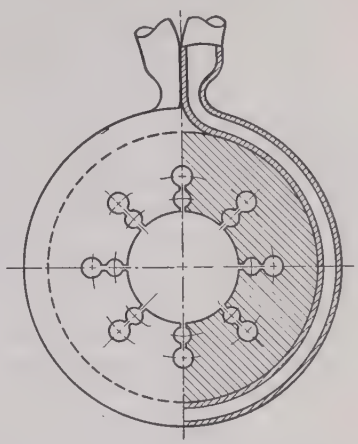

Fig. 2—Arrangement of an anode with single-cavity cells in combination.

The ratio of d_a/d_k should not be made smaller than a certain value. The diameter of a unit resonant anode cavity is expressed by d_a. For a single-cavity anode, it should not be made lower than 1.3. For a four-cavity anode, it may be as low as 1.1. If a decrease in this ratio below the above values were made, there resulted an abrupt drop in efficiency. The value of this limiting ratio for each form of anode depends upon other features of the design. At the same wavelength, in a four-cavity anode, larger dimensions of both d_a and d_k are permissible and permit production of increased power outputs.

IV. Production of High-Power Oscillations

Characteristic performance data for the one-, two-, and four-cavity anodes are shown in Table I.

TABLE I

Test Number	d_a/d_k	E_b Volts	I_b Milliamperes	H Gauss	λ Wavelength in Centimeters	$H\lambda$	Output Watts P	Per cent Efficiency	Remarks
1	1.5	1700	140	2600	7.7	20000	≅8	≅3	Anode shape of Fig. 3.
2	2.0	1800	45	2000	9.9	19800	7	9	Anode shape of Fig. 4 at low loading.
3	1.5	2650	360	1350	9.0	12100	170	18	Anode shape of Fig. 5.
4	1.5	4400	330	1950	9.0	17500	300	20	Same tube but higher H and E_b. Serious back heating.
5	1.5	3220	160	1650	9.1	15000	116	22.5	Same anode but in a sealed-off tube.

It is to be noted that the two-cavity magnetron whose performance is given in test 2 of Table I showed considerably higher efficiency than the results on the single-cavity magnetron given in test 1. It is to be noted, however, that its power output was lower which resulted

from the fact that the anode dimensions were not optimum. (See Figs. 3 and 4.)

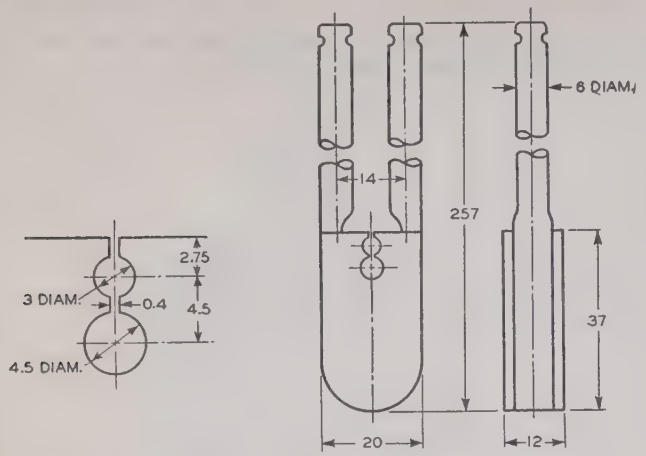

Fig. 3—Single-cavity anode of a 7.7-centimeter magnetron. All dimensions are in millimeters.

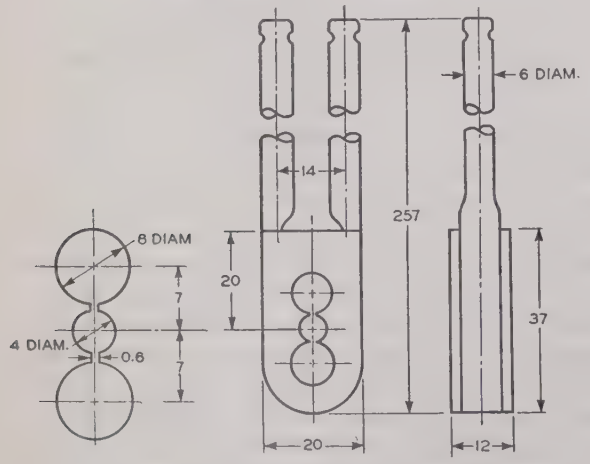

Fig. 4—Double-cavity anode of a 9.9-centimeter magnetron. All dimensions are in millimeters.

It will be noted that the figures for tests 3 and 4 of Table I describing results with the four-cavity magnetron showed both higher outputs and efficiencies than those obtainable from the single- and two-cavity anodes. (See Fig. 5.)

The condition of operation recorded for test 4 was characterized by increased magnetic field and this resulted in extremely strong back heating (cathode conflagration) of the cathode. This back heating or increase of cathode temperature resulting from the oscillation of the tube was the chief cause for inability to go beyond a 300-watt output in spite of the ample anode-cooling properties of the design.

Another factor limiting the power output was the limitation of the high-voltage power supply. This latter limitation forced us to use extremely low series ballast resistors in the anode to cathode circuit. This rendered the cathode liable to destruction. With power outputs in the neighborhood of 100 watts at a wavelength of 9 centimeters, it was possible to obtain high-frequency brush discharges into the air 5 millimeters long from the end of a piece of wire 0.33 millimeter in diameter.

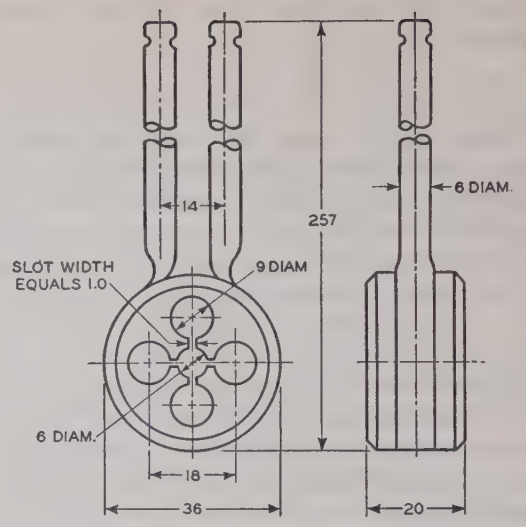

Fig. 5—A four-cavity anode of a 9.0-centimeter magnetron. All dimensions are in millimeters.

A resonator made of 0.5-millimeter nichrome wire became red hot if properly connected to the output of the magnetron when oscillated at a 60-watt output.

Textolite type of insulation at such powers and frequencies was entirely unsuitable for use as it caught on fire as soon as touched by the output leads or the resonant output wires.

Several sealed-off magnetrons were made with anodes of the same general design. They operated continuously and showed a stable output of 100 watts. The operating condition for a typical sample is recorded as test 5 in Table I.

Variation of power output with power input for a typical sealed-off magnetron is shown by the curve of Fig. 6. In this case, the direct anode supply voltage was

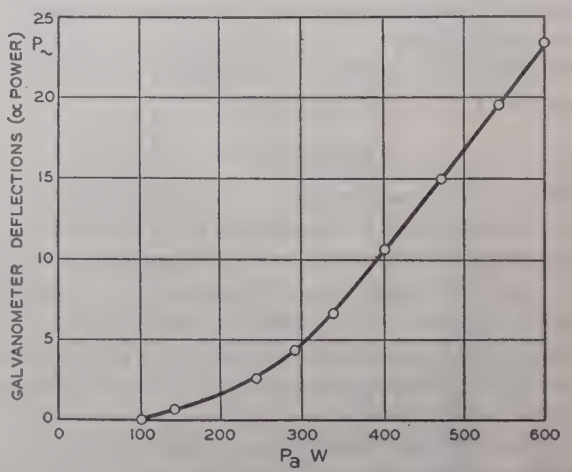

Fig. 6—Power output versus power input of a sealed magnetron.
Abscissa: Input watts.
Ordinate: Relative power output.

held constant so that the change of input power resulted entirely from a change of input current.

V. Magnetrons for Outputs at Still Shorter Wavelengths under Conditions When $n<1$

With four-cavity anodes of similar design but smaller proportionate size than those which have been described much higher frequency oscillations were obtained. Table II lists the experimental data obtained from two such magnetrons of the demountable type.

TABLE II

Test Number	d_a/d_k	E_b Volts	I_b Milliamperes	H Gauss	λ Wavelength in Centimeters	$H\lambda$	Output Watts P	Per cent Efficiency
1	1.4	1570	190	1600	5.5	8800	20	7.0
2	1.4	890	48	2560	2.6	6650	2	4.5

The magnitude of the product of $H\lambda$, shown in this table for the case where $n=\frac{1}{2}$, was approximately one half that necessary for oscillations of the first order where $n=1$.

A comparison of the operating conditions for two modes of oscillation in an identical magnetron is shown in Table III. The anode of this latter magnetron was of the four-cavity type having a diameter for the accommodation of the cathode of 4.4 millimeters, the diameter of each individual anode cavity being 6.8 millimeters. The width of the slot was 0.4 millimeter. Operating results, under conditions where $n=\frac{1}{2}$, were critical, this being particularly true for changes in the strength and direction of the magnetic field. However, efficiencies as high as 8 per cent were observed.

TABLE III

Oscillation Mode $n=$	E_b Volts	I_b Milliamperes	H Gauss	λ Wavelength in centimeters	$H\lambda$
1	2600	208	2160	7.6	16,400
$\frac{1}{2}$	1000	118	1150	7.5	8,600

In order to obtain oscillations at still shorter wavelengths in the centimeter-wave band, this mode of oscillation where $n=\frac{1}{2}$ would be of great advantage since the magnetic field required would be less than for the case where $n=1$ and this would result in reduced cathode back heating. This effect was found to be the chief obstacle in obtaining higher power outputs at these frequencies. However, there are a number of questions requiring further study for obtaining oscillations of shorter wavelengths using this mode.

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Paper Capacitors under Direct Voltages*

M. BROTHERTON†, nonmember, I.R.E.

Summary—**This article discusses and illustrates the influence of voltage, temperature, and materials on the life of paper capacitors under direct voltages. The life decreases as the applied voltage and ambient temperature increase. Operating voltages which are safe at room temperatures may produce rapid dielectric failure at high temperatures unless the most suitable materials and best manufacturing practices are employed. It is essential that temperature as well as voltage be taken into account in the design, manufacture, and use of paper capacitors if trouble-free service is to be insured. Paper capacitors should be rated for a maximum direct operating voltage at a maximum ambient temperature. An accelerated life test on representative samples is the best criterion of the life performance that may be expected of a manufactured lot of capacitors in service.**

This article also describes some types of asphalt-sealed and hermetically sealed paper capacitors designed for direct-current operation in different types of service.

PROBABLY no element is more familiar in telephone and radio apparatus than the electric condenser or capacitor. It serves a variety of purposes: providing negative reactance, storing electric charge, passing alternating current, or blocking direct current. Technically, it provides capacitance, one of the three basic parameters of an electric circuit.

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Most widely used is the impregnated-paper type since it provides capacitance in compact form at relatively low cost for all types of service where the high precision and stability and low losses of mica capacitors are not essential. Where used for filtering, blocking, and by-pass purposes in radio or other communication equipment, the principal requirement is that they shall withstand

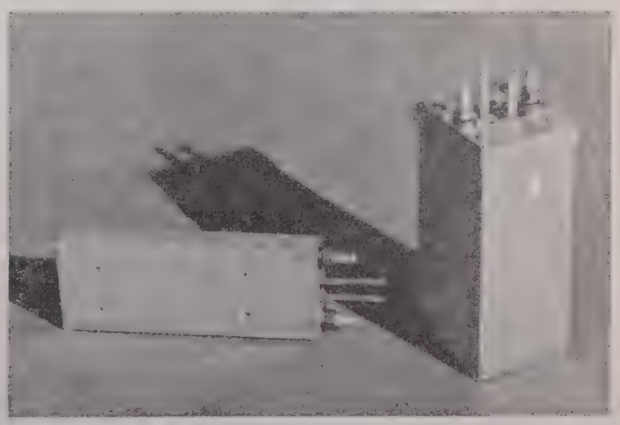

Fig. 1—Asphalt-sealed paper capacitors.

direct voltage over the service life of the equipment. Paper capacitors are, however, limited in their ability to withstand sustained direct voltages especially at high temperatures and this must be carefully taken into account by designers, manufacturers, and users of these capacitors to insure satisfactory service. These limitations are becoming increasingly important with the pressure on designers of communication apparatus to accommodate more power in less space, resulting in higher apparatus temperatures.

This article discusses the factors limiting the direct-current life of paper capacitors. It also describes some types of paper capacitors developed by the Bell Telephone Laboratories and manufactured by the Western Electric Company primarily for direct-current operation.

Paper Capacitors for Room Temperatures

The most widely used paper capacitor in the Bell System is the asphalt-sealed type, Fig. 1, which houses the capacitor unit in a metal can sealed with asphaltic compound. The one on the left has a capacitance of 1 microfarad and the one at the right, 4 microfarads. Simplicity of design coupled with large production makes these capacitors small in size as well as low in cost. So many of them are now in use in the Bell System that the strips of paper in them, if fastened end to end, would extend to the moon and back with enough left over to go five times around the world.

The capacitance is provided by a unit consisting of two strips of tin or aluminum foil separated by at least two thicknesses of paper, as shown in Fig. 2. The interleaved paper and foil are rolled together, dried, and impregnated under compression to form a compact unit. It is not generally realized how thin the paper insulation between foils must be made to secure the small size desirable. A 1-microfarad unit capable of withstanding direct-current potentials up to 200 volts is smaller than a five-cent bar of chocolate. To accomplish this, the paper between foils must be somewhat less than a thousandth of an inch thick, and must be worked at approximately ninety times the voltage gradient on lamp cord in everyday household use. A 1-microfarad paper-capacitor unit designed to operate under the low-voltage gradient used in lamp cord would be as large as a suitcase.

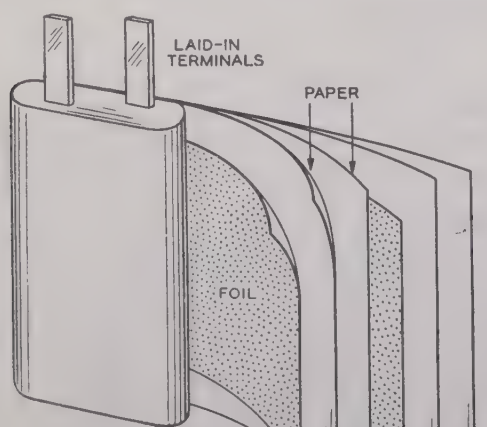

Fig. 2—Typical rolled-paper capacitor unit with "laid-in" terminals.

The ability of the impregnated paper to withstand these severe voltage gradients has been made possible by minimizing the amount of chemically active materials such as acids, alkalies, and water in the paper and impregnant. These agents react in an electric field, and if present even in small amounts may result in rapid degeneration of the impregnated paper until dielectric failure occurs. Because of this high-voltage gradient, traces of impurities that are of little consequence in lamp-cord insulation would lead to rapid failure in paper capacitors.

Asphalt-sealed capacitors are intended, in general, for indoor conditions at ambient temperatures from 16 to 50 degrees centigrade; and under these conditions can be made to perform satisfactorily for long periods with ratings up to 200 volts. Waxes of the chlorinated and hydrocarbon types are permissible provided the other materials are of requisite purity and dryness. They are, however, limited in the temperatures at which they may be reliably operated. At temperatures above 50 degrees centigrade, some of the asphaltic sealing compound may drain from the container and expose the unit to the infiltration of moisture, and also some of the compound may migrate into the unit, causing destructive chemical action. Nor are they suited to extreme cold which may crack the protective jacket of the compound and expose the unit to moisture.

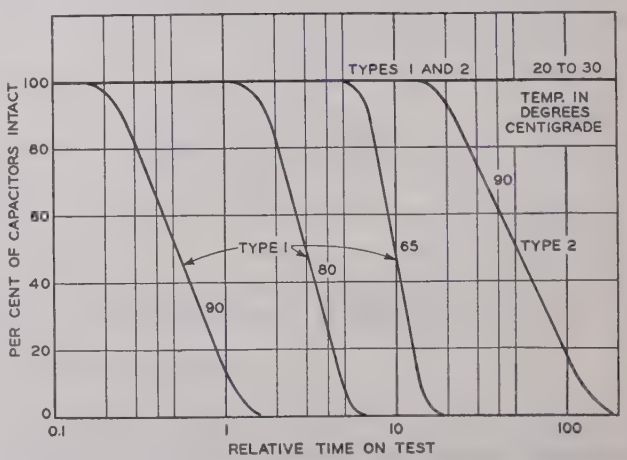

Fig. 3—Cumulative-failure distribution curve showing effect of temperature at constant direct voltage.

Paper Capacitors for Wide Temperature Ranges

In capacitors operating considerably above 50 degrees centigrade or below 0 degrees centigrade, it is not only essential to employ superior means of sealing but it is also desirable to choose oils rather than waxes as impregnants. This avoids the limitation on ratings imposed by occurrence of solid to liquid transformations in region

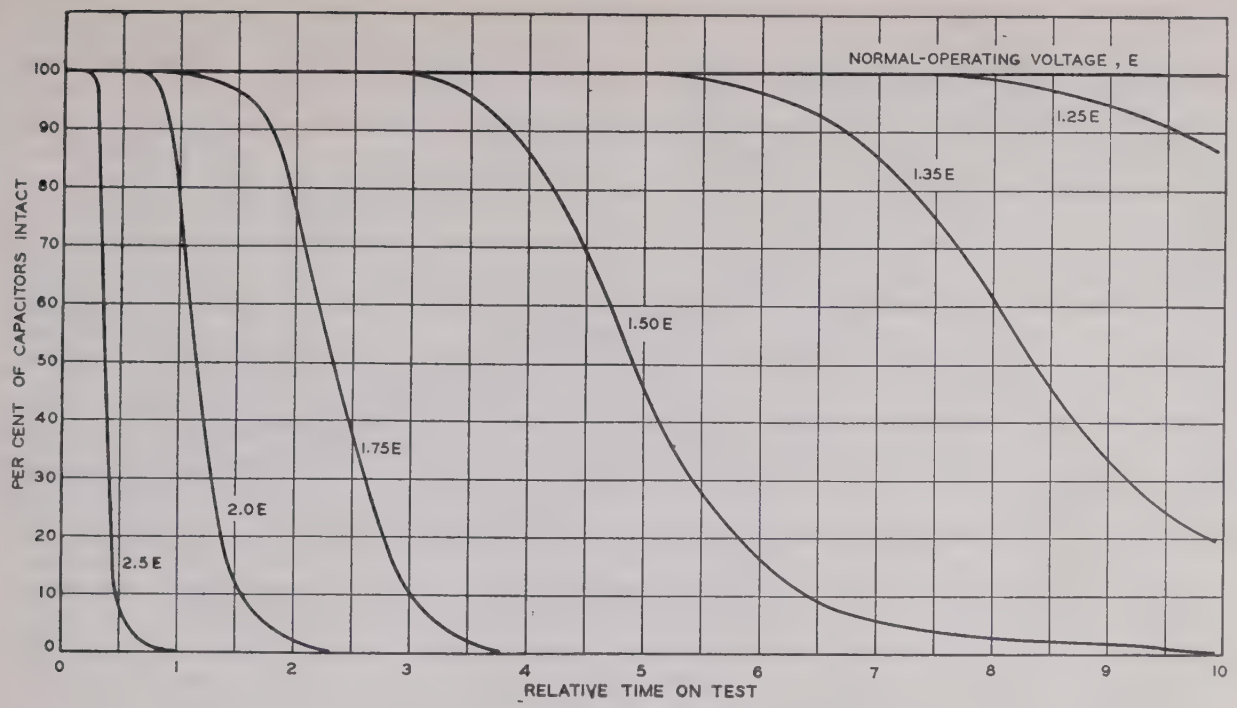

Fig. 4—Cumulative-failure distribution curve showing effect of direct voltage at constant capacitor temperature.

of softening points of waxes. It also avoids the greater tendency to dielectric failure exhibited by wax-impregnated capacitors at temperatures below 0 degrees centigrade. Beyond these measures, it is vital to limit the choice of combination of paper, foil, and impregnant to those that are least sensitive to chemical deterioration.

The importance of the latter factor is brought out by Fig. 3, which depicts test results on groups of oil-impregnated capacitors (type 1) using materials that yield thoroughly satisfactory performance at temperatures of 20 to 30 degrees centigrade. In this instance the life at 90 degrees centigrade is only a small fraction of the life at room temperature. From an operating standpoint this means that the voltage at which this type of capacitor may be safely operated at 90 degrees centigrade is considerably less than the safe voltage at room temperatures. The destructive effect of heat varies widely for different types of paper, impregnant, and foil materials and also depends on the amount of water or other contaminants present.

There are other materials which are much less sensitive to deterioration in this same higher range of temperature. They generally follow much the same pattern except that the time scale for a given voltage is increased many fold. Fig. 3 also illustrates the life at 90 degrees centigrade for capacitors (type 2) using materials better suited to withstand high temperatures. At 90 degrees centigrade the life of type 2 is approximately 100 times the life of type 1 under the same voltage.

The life of a paper capacitor under sustained direct voltage decreases rapidly as the voltages increases. Where a group of capacitors which have passed a suitable dielectric strength test is subjected to a sustained direct voltage, there is an initial period during which no significant failures[1] occur. (See Fig. 3.) At the end of this period there is a definite inflection in the failure distribution curve where significant failures start and thereafter continue to occur according to a definite pattern. This initial period which we shall denote as "L" represents the minimum life to be expected of the group under the specific applied voltage. It is this minimum life, rather than the average or maximum, which is of primary interest from the standpoint of insuring trouble-free capacitor operation. It has been found experimentally that "L" varies approximately as $1/E^n$ when "E" denotes the direct-current potential across the capacitor terminals. The value of "n" has been found experimentally to range from 4 to 6 for capacitor impregnants in general use at the present time, and on the basis of capacitance values up to 4 microfarads, ambient temperatures up to 85 degrees centigrade, test voltages up to 3000 volts, potential gradients up to approximately 1500 volts per mil of impregnated paper dielectric, and provided the internally generated heat due to direct currents is small.

It has been found to apply to liquid impregnants and also to waxes from room temperature up to near the melting point of the wax. There are indications that it does not apply where failure is apparently complicated by causes other than progressive deterioration of the dielectric attributable to externally applied heat and voltage; for example, with some wax-type impregnants

[1] "That is, ignoring the small percentage of capacitors of any test group which may fail early in the test due to random dielectric weaknesses, especially where the test voltage is appreciably above normal operating values."

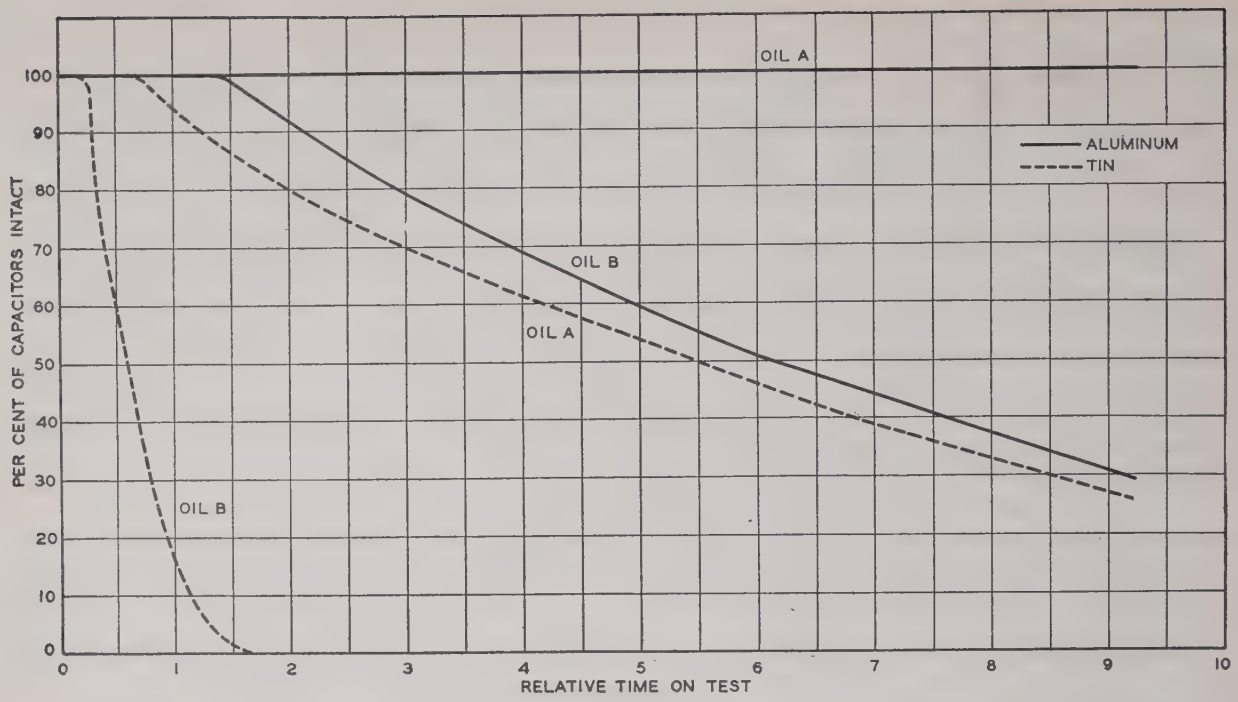

Fig. 5—Cumulative-failure distribution curve at 80 degrees centigrade.
1. Oil *A* versus oil *B*.
2. Aluminum-foil versus tin-foil capacitors.

which show a tendency to fail under direct-current potentials during temperature swings below room temperature or during the change from solid to liquid at the melting point of the wax.

This empirical formula for life versus voltage provides a valuable working basis for determining safe operating voltages by means of accelerated life tests. Fig. 4 shows cumulative-failure distribution curves at different voltages plotted on the basis of "n" = 5. With reference to Fig. 4, suppose a particular design of paper capacitor will be required to operate under a direct voltage (E) at ambient temperatures which may reach 90 degrees centigrade. Also, suppose that an adequate number of samples representing this capacitor show a life greater than 10 days under a voltage of 2.5E or 30 days at 2.0E at a sustained temperature of 90 degrees centigrade. It follows that this capacitor could be expected to have a life in excess of 1000 days when operated at the expected operating voltage (E) with the temperature sustained continuously at 90 degrees centigrade. Furthermore, a service life considerably in excess of 1000 days could be expected of this capacitor where, as in most types of service, the capacitor would not be operated 24 hours per day or continuously at the maximum temperature while under voltage. In some applications, such as telephone repeaters and radio broadcast stations, capacitors may have to withstand voltage in combination with high ambient temperatures continuously. In engineering capacitor designs for such applications similar short-time accelerated tests can also be used with due attention, of course, to the fact that 10- to 15-year life may be required.

Experience shows that no other single test provides an equal assurance of providing capacitors which will stand up in service than an accelerated life test on representative samples. It is most desirable that capacitors which involve radically new design, new materials, new sources of supply for materials, or are required to meet

Fig. 6—Hermetically sealed paper capacitors.

service conditions not hitherto encountered should be rated only after life-testing under accelerated voltages and in combination with temperature conditions at least as severe as those in view.

Also, since the direct-voltage life of paper capacitors decreases as both temperature and voltage increase, the best practice dictates that they should be rated for a maximum direct operating voltage at a maximum ambient temperature.

As previously indicated the quality and type of materials used for the impregnant and foil electrodes greatly affect life, especially at high temperatures. Fig. 5 shows comparative life performance for capacitors impregnated with two mineral oils which are used as capacitor impregnants. Oil *A* provides considerably longer life than oil *B* and aluminum foil is better than tin foil as an electrode material with either oil. The importance of the electrode material also applies to other impregnants such as chlorinated diphenyl or castor oil. Consequently, the use of aluminum foil is preferred in capacitors intended for high-temperature operation.

Since deterioration of the dielectric is accelerated in the presence of water even where the best materials are used, it is essential not only that the units be well dried initially but also that the casing used completely protect the capacitor units from moisture in service. This consideration is especially important in equipments for the Armed Forces which are subject to vibration and shock in combination with extreme climatic conditions. This seal should remain airtight over an extremely wide range of temperature, and under severe mechanical shock.

Fig. 6 shows two types of capacitors which are housed in containers with such hermetic seals. The capacitor shown at the left in Fig. 6 is filled and impregnated with a hydrocarbon wax; the terminals are brought out through neoprene-treated rubber-insulated leads and the hermetic seal is secured by a metal sleeve which is constricted on the rubber insulation and soldered to the container. They are widely used for by-pass and filtering purposes in carrier telephone, public address, and radio equipments for temperatures extending from above freezing to 65 degrees centigrade. Below the freezing point, their effectiveness is limited because they must be worked below normal rated voltage to secure satisfactory life.

The capacitor shown at the right has its leads brought out through molded phenol plastic and it is impregnated and filled with chlorinated diphenyl which is a heavy liquid. For the same capacitance and voltage rating, the type of capacitor shown at the right in Fig. 6 is considerably smaller than that shown at the left, because of the high dielectric constant of chlorinated diphenyl as compared with wax. Furthermore, they may be operated at temperatures from −50 to +85 degrees centigrade without damage, and they are much superior to wax-type capacitors for operation at high alternating-current 60-cycle voltages. The chlorinated-diphenyl type has the disadvantage, however, that its capacitance decreases at subzero temperatures by an amount which is intolerable where high stability of capacitance is an essential requirement. Its small size is advantageous where it is necessary to accommodate large lumps of capacitance for blocking, filtering, and by-pass purposes where large variations in capacitance can be tolerated. The capacitor illustrated contains a stabilizing agent in the chlorinated diphenyl which materially increases the direct-voltage life of the capacitor especially when operated at temperatures in excess[2] of 50 degrees centigrade.

For reasons of inability to withstand high and low temperatures or because of too great a variation of capacitance with temperature, none of the three types of capacitors discussed above is suitable where it is essential to secure a small capacitance change in apparatus operated over a temperature range extending from −40 to +85 degrees centigrade. To meet this requirement, a fourth type is provided which is impregnated and filled with a heavy mineral oil. This type has a very long direct-voltage life at high temperatures as well as low. Its total capacitance variation may be limited to 6 per cent over the above-mentioned temperature range at a frequency of 1000 cycles. This capacitor is housed in sealed containers similar in construction to those of Fig. 6. As regards size, mineral-oil capacitors are about the same as the hydrocarbon-wax capacitors and are, therefore, generally larger than the chlorinated-diphenyl type for a specified capacitance and voltage rating.

Today, large quantities of paper capacitors are being used by the Armed Forces in radio and other communication equipment. Such equipment is required to operate reliably under an exacting variety of climatic conditions. The success or failure of military operations may hinge on the dependability of communication equipment which may be rendered temporarily useless at a critical moment by the failure of a single paper capacitor. Accordingly, it is especially important that designers, manufacturers, and users of radio and other communication equipment for the Armed Forces should take into account the influence of operating voltage, ambient temperature, and of materials on the life of paper capacitors under direct voltages.

[2] D. A. McLean, L. Egerton, G. T. Kohman, and M. Brotherton, "Paper dielectrics containing chlorinated impregnants, *Indus. and Eng. Chem.*, vol. 34, pp. 101–109; January, 1942.

Vacuum-Tube Networks*

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Summary—**The performance characteristics of vacuum-tube amplifiers are analyzed by combining the fundamental relations governing the motions of electrons within the vacuum tube with the methods of circuit-network theory. The result is an equivalent network based upon the electron-discharge stream rather than upon the external terminals of the tube. It is connected to the external terminals through simple impedance elements and allows the amplifier performance to be calculated in a comparatively straightforward manner even in the case of multielement tubes and when the electron transit time is not restricted to a small portion of the cycle. The phase delay in the transmission which results from electron transit time is calculated together with the input loading. This calculated loading must be increased to include the effects of Maxwellian distribution of electrons, which may be disregarded for a first approximation in many other applications. The analysis methods are applicable to velocity variation devices as well as to density variation or space-charge control and methods of handling such problems are briefly illustrated.**

PRINCIPAL SYMBOLS

Fundamental Constants

Symbol	Meaning	First Appearance (Eq.)
e	Electron charge, 1.59×10^{-19} coulomb	(1)
m	Electron mass, 9.03×10^{-28} gram	(1)
η	Constant ($=10^7\ e/m$)	(1)
ϵ	Permittivity of vacuum ($=10^{-11}/36\pi$)	(2)
i	Imaginary unit ($=\sqrt{-1}$)	(6)

General Symbols

Symbol	Meaning	First Appearance (Eq.)
V_D	Direct-current potential, volts	(1)
V	Alternating-current potential, volts	(5)
I_D	Direct current, amperes	(2)
I_m	Maximum direct current, amperes	(3)
ζ	Space-charge factor	(2)
I	Alternating current, amperes	(5)
q	Alternating conduction current, amperes	(5)
v	Alternating electron velocity, centimeters per second	(5)
u	Direct-current velocity, centimeters per second	(1)
x	Distance, centimeters	(2)
ω	Angular frequency, radians per second	(6)
T	Electron transit time, seconds	(2)
θ	ωT, electron transit angle, radians	(6)
β	$i\theta$, complex transit angle	(6)
$A^*, B^*, C^* \cdots I^*$	Electronic coefficients	(5)
P, Q, S	Electronic coefficients	Table I
r_0	Zero-frequency diode resistance, ohms	(13)
g_0	Zero-frequency diode conductance, mhos	(26)
C	Capacitance, farads	(9)
y_{11}, y_{22}, y_{33}	Self-admittances of regions 1, 2, 3	(21)
y_{12}, y_{13}	Transadmittances from region 1 to regions 2 or 3, respectively	(21)
y_{23}	Transadmittance from region 2 to region 3	(21)
α	Grid capture factor	(20)
ϕ	Phase angle of a transadmittance, radians	(28)
g_m	Control grid-plate transconductance, mhos	(56)
y_m	Control grid-plate transadmittance, mhos	(58)
Y_{in}	Input admittance, mhos	(60)
Z_l	Impedance external to a vacuum tube, ohms	(55)
Y_l	Admittance external to a vacuum tube, mhos	(54)
$z=r+ix$	Impedance internal to a vacuum tube, ohms	(11)

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WITH the increase of knowledge concerning the electronics of current flow inside of vacuum tubes, which has taken place during recent years, it now appears timely to review the situation and to arrange the resultant equations in a way better suited for physical interpretation and application. In doing this, it has been found that several properties of vacuum tubes may be demonstrated readily by selecting equivalent networks to represent the performance of the vacuum tube and its attached circuits in such a way that the equivalent network conforms most simply with the equations expressing the tube performance rather than with the physical configuration of the tubes as has been the custom heretofore. For example, the present conventional-network representation of a three-element vacuum tube is based upon the three available terminals external to the glass envelope housing the electrodes of the tube and is drawn to represent the equivalent impedances which would be measured looking into the various pairs of terminals thus exposed. The resulting network is in a convenient form to use when the impedances so obtained are simple in nature. This is the case with triodes operating at low frequencies. It is true to a lesser extent with tetrodes and pentodes operating at low frequencies but it is not true at all with tubes operating at higher frequencies or when a more searching analysis of multielement tubes, even at low frequencies, is desired.

The mathematical attack on the problem of tube performance proceeds from a somewhat different viewpoint and the equations are set up in such a way that attention is centered on the beam of electrons flowing through the tube. The equations have been arranged in a fairly compact form[1] and the attempt has been made to transform them by straightforward mathematical manipulation into such a form that they coincide with the preconceived idea of the equivalent network of the tube based upon the hypothesis that it should be built around the three or more available external connections to the tube. The result was a network which, at moderate frequencies and for triodes, was a more or less minor modification of our older network, and has been discussed in previous papers.[1,2] When the procedure was applied to tetrodes and pentodes or when the triode network was extended to higher frequencies, the network

[1] F. B. Llewellyn, "Electron Inertia Effects," Cambridge University Press, New York, N. Y., 1939.

[2] F. B. Llewellyn, "Equivalent networks of negative grid vacuum tubes at ultra-high-frequencies," *Bell Sys. Tech. Jour.*, vol. 15, pp. 575–586; October, 1936.

was more unsatisfactory. This led to the suspicion that the network constructed upon the principle of equivalent impedances between the three or more external terminals to the tube is not the most convenient network to use in the general case. Accordingly, it was abandoned entirely and the simplest network based upon the mathematical equations of the tube performance was sought, regardless of its relation to the external or apparent geometry of the tube.

The result is a network which is much simpler than the former one for many purposes. Its use requires a little readjustment of preconceived ideas concerning the inner workings of vacuum tubes and difficulty in attempting to explain it in a useful manner arises from the fact that it represents a combination of two techniques which at present are largely dealt with by two different groups of people without too much co-ordination between them. We have to employ what is generally called "electronics analysis." The people who handle this type of analysis are concerned with the functioning of the inside of the vacuum tube, with the paths or trajectories of the electrons, and with the forces acting upon them. They express their result in terms of a well-known quantity called the "transconductance" of the vacuum tube in conjunction with the interelectrode capacitances. Having done this, the job of the electronics engineers is completed and the result is turned over to the other group, which is composed of circuit people. Taking the data supplied by the electronics group, these circuit people apply the principles of circuit analysis to determine the performance of the given tube when it is connected to circuit arrangements of inductance, capacitance, and resistance in various configurations. Their techniques and methods differ widely from those of the electronics group.

In the present analysis both of these techniques have to be combined and intermingled in a manner that will require a considerable readjustment of viewpoint by both groups. However, the resulting network and physical ideas upon which it is based have already proved so helpful in analyzing the performance of tubes that it is felt that the time spent in a thorough explanation of the basic principles and a carrying out of the consequent analysis will be well repaid.

Another situation that is unfortunate for the time being, but may eventually prove helpful, is the fact that the new network is most conveniently based on the "nodal" rather than on the "mesh" form of circuit analysis and that the nodal analysis is less widely used and is therefore less familiar than the mesh analysis. Moreover, the network has some of its nodes located in the interior of the vacuum tube rather than coinciding with its external terminals. Specifically, the nodes are taken to coincide in position with the planes of the various grids within the tube but the nodal potentials are not the same as those of the grid wires themselves but correspond to what the electronics people call the "effective potential" of the grid in question.

The reason why this is the normal and natural way to select the nodes for the circuit analysis may be seen by starting from the electronics analysis and discussing its basic principles. To do this we must review briefly what has been published in several preceding papers.[1–7]

First of all there is the concept of total current as distinguished from its components of conduction[8] current and displacement current. The total current has the all-important property that it always flows in closed paths so that if we were to select a fictitious tube or cylinder whose bounding walls coincided in space with the direction of current flow, then the total current flowing into one end of that tube would be exactly equal in every respect to the total current flowing out of the other end at the same instant of time. For example, if we have two parallel planes of practically infinite extent and select condition so that our tube is a right circular cylinder with its axis perpendicular to the two given planes, then the total current flowing into one end of the cylinder is exactly equal at every instant to the total current flowing out at the other end.

This seems to be a simple and straightforward concept but is likely to cause difficulty when the two components of the total current, namely, conduction and displacement current, are confused with the total current. To see this, suppose that electrons are injected uniformly for an instant perpendicularly through one of the two planes which cap the cylinder described above, and move across to the other plane. The electrons take a finite length of time to complete the path. At the instant that they cross the first plane, there is a certain current through that plane into, or out of, the corresponding end of the cylinder. According to the properties of total current described above, precisely and exactly that same current flows out at the other end of the cylinder at the same instant. But, you say, how can this be when the electrons constitute a current and the electrons are moving only through the first surface?

The answer, of course, is that the flow of electrons does not constitute the total current but only the conduction component of the total current and that the remaining component, consisting of displacement current, exactly fills up the gap between the current carried by the electrons themselves and that required to make the total current flowing through both ends of the cylinder identical.

This concept cannot be emphasized too strongly and

[1] W. E. Benham, "Theory of the internal action of thermionic systems at moderately high frequencies," *Phil. Mag.*, vol. 5, pp. 641–662; March, 1928; and vol. 11, pp. 457–517; February, 1931.

[4] J. Müller, "Electronenschwingungen im Hochvakuum," *Hochfrequenz. und Electroakustik*, vol. 41, pp. 156–167; May, 1933.

[5] F. B. Llewellyn, "Operation of ultra-high-frequency vacuum tubes," *Bell Sys. Tech. Jour.*, vol. 14, pp. 632–665; October, 1935.

[6] C. E. Fay, A. L. Samuel, W. Shockley, "On the theory of space charge between parallel plane electrodes," *Bell Sys. Tech. Jour.*, vol. 17, pp. 49–79; January, 1938.

[7] L. C. Peterson, "Impedance properties of electron streams," *Bell Sys. Tech. Jour.*, vol. 18, pp. 465–481; July, 1939.

[8] Some writers use the term "convection" current for this quantity. The meaning employed here will be clear from subsequent discussions.

the understanding of the properties of vacuum tubes with their interpretation in terms of geometrical figures depends upon its thorough appreciation.

Basic Electronic Principles

The basic configuration for the mathematical formulation of vacuum-tube electronics is the flow of electrons between parallel planes. While it is true that a concentric-cylinder arrangement is more suitable for some of our tubes, the mathematics for cylinders is so much more complicated than for planes, that the advantages of direct application to cylindrical tubes is more than counterbalanced by the ease of manipulation in the parallel case. Moreover, there is a tendency apparent already toward making more of our tubes in the plane-parallel form and fewer in the cylindrical. This is because of the practical advantages to be gained from a uniformly dense electron flow in comparison with a converging or diverging one, and not because of ease of solving mathematical equations. The tendency is a fortunate one, however, for it allows direct application of the equations to be made to an increasingly greater number of tubes, and leaves the qualitative interpretation of the equations as sufficient foundation to form a basis of comparison for the performance of the other geometrical shapes.

The basic picture upon which the analysis is built is shown in Fig. 1. The two parallel planes between which the electron flow takes place are marked a and b, and the flow is assumed to occur from left to right. The planes need not coincide with any particular electrodes of an actual vacuum tube, but may be located at will, subject only to the restrictions that the electron flow is perpendicular to the planes, is essentially uniform over their surfaces, and that no electrodes of the actual tube are located between them.

Strictly, the analysis is confined to consideration of electrons initially emitted from a thermionic cathode with a single value of velocity rather than with the Maxwellian spread which actually exists. Also, the electrons must move from left to right only in Fig. 1; never from right to left. Cases where departures from these restrictions modify the conclusions are pointed out in several places in the following pages.

As a given condition of the problem, a certain total current[9] per unit area is assumed to flow perpendicularly between the planes. The electron flow being from left to right, it is appropriate to take the total current as flowing from right to left. In general, it consists of a constant component I_D and a number of alternating components which may be designated by the general letter I, without any subscript. The total current per unit area is thus I_D+I. At any instant it has the same value at both planes a and b regardless of whether more electrons are passing through one plane than the other at that instant.

[9] Throughout this paper, the rationalized system of *MKS* units is used, so that lengths are in centimeters while electrical quantities are in the practical units of volts, amperes, ohms.

Another given condition of the problem involves the velocities of the electrons. Just as the total current was separated into direct and alternating components, so also may the electron velocity be separated into direct and alternating components. One of these components, corresponding to the direct current, has a constant value for all time at a given plane, a or b. It may be expressed immediately in terms of the direct-current potential on that plane, by means of the equation

$$V_D e = 10^{-7} m u^2/2 \quad \text{or} \quad \eta V_D = u^2/2 \tag{1}$$

where V_D is the direct-current potential in volts, u is the direct-current electron velocity in centimeters per second, and $\eta = 10^7\, e/m = 1.76 \times 10^{15}$. In much of the following analysis, it is simpler to deal with the direct-current electron velocity rather than the direct-current potential, but the one may always be expressed in terms of the other by use of (1).

The alternating-current velocity v is a little harder to understand. Its significance may be explained if we imagine ourselves to be stationed at a given point in space and make a record of the time at which each electron passes us together with the velocity of the electron

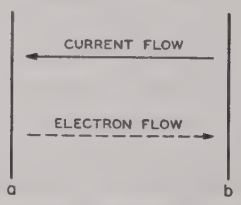

Fig. 1—Basic picture for electronic analysis.

at that instant. When these data are plotted with time as ordinate and velocity as abscissa, the points may be connected by a smooth curve showing velocity as a function of time. The average abscissa then gives the direct-current velocity and the deviation from the average gives the alternating-current velocity. Unlike the direct-current velocity, the alternating-current velocity cannot be expressed in terms of the direct-current potential by a simple relation such as (1). This distinction should be kept in mind, as it is very important. The velocity is a unique measure of the kinetic energy of the electron, but is not so simply related to the alternating-current potential or voltage except at such low frequencies that the voltage between the a plane and the b plane in Fig. 1 does not change appreciably while the electron is passing between them. The total velocity at a given instant at the a plane may be written u_a+v_a while u_b+v_b expresses its value at the b plane.

The third, and last, given condition is the conduction current. Its density at a given plane is the product of charge density and electron velocity. Like the total current, it may be separated into direct and alternating components. The direct component is the same as I_D, the direct component of the total current. The alternating component will be represented by the symbol q, so that the conduction current per unit area at the a plane in Fig. 1 is I_D+q_a while at the b plane it is I_D+q_b.

With these notations, the electronics analysis expresses conditions between the a plane and the b plane by means of two sets of equations; the one set for direct-current and the other set for alternating-current. It is true that distortion effects are not included in these equations and require further sets of equations for their analysis. This, however, is exactly analogous to the situation with which we have had to cope ever since the non-linear properties of vacuum tubes were first analyzed, and should cause no confusion in the mind of the circuit engineer.

The first set of equations, then, may be written as follows:

$$\left.\begin{aligned} \zeta &= 3(1 - T_0/T) \\ x &= (1 - \zeta/3)(u_a + u_b)T/2 \\ (\eta/\epsilon)I_D &= (u_a + u_b)2\zeta/T^2 \end{aligned}\right\} \qquad (2)$$

where $\eta = 1.76 \times 10^{15}$ has been encountered in connection with (1), the factor $\epsilon = 1/(36\pi \times 10^{11})$ is the permittivity of vacuum, $\eta/\epsilon \doteq 2 \times 10^{28}$, the distance x is measured in centimeters from the a plane to the b plane, T is the time it takes an electron to traverse that distance, and I_D is the current density in amperes per square centimeter. The reference time T_0 and the space-charge factor ζ require further explanation, as follows:

With reference to Fig. 1, and under the conditions that one and only one electron were introduced into the space between the two planes, the force acting on the electron would be a constant regardless of its position. The motion of the electron could therefore be calculated very easily and the time required for it to move from the a plane to the b plane may be found from the expression $x = (u_a + u_b)T_0/2$. Comparing this with the second of equations (2) we see that T_0 is the value which T would approach when ζ approaches zero. That is, T_0 is the transit time when there are no other electrons present between the two planes besides the one under observation. This absence of other electrons is called the condition of zero space charge.

In actual vacuum tubes there are usually many electrons present between the two planes at any given instant. Their presence modifies the force acting on any given electron, and the space-charge factor ζ is a measure of the effectiveness of that modification. In the usual treatment[6] of the direct-current space-charge problem the solutions of the fundamental equations are obtained in terms of parameters which are difficult to apply directly to the problem at hand. For this reason, it is expedient to rewrite the solutions in terms of direct-current transit times as a parameter. Such a procedure allows the degree of space charge to be specified quantitatively by defining a space-charge factor, which we call ζ. As expressed in (2), it is zero when there is no space charge. As more and more electrons are injected through the a plane and move across to the b plane, the density of the space charge increases and ζ increases likewise. However, it is a well-known fact that the amount of electron current which may be injected through the a plane and that will thereafter move across to the b plane is not unlimited, but has an upper value beyond which it is impossible to force more electrons into the space without having some of them turn around and move backwards toward the a plane with a consequent reduction in the number crossing the b plane. The onset of this phenomenon occurs very suddenly for a critical value of the injected current which we shall call I_m. When that value of injected current is exceeded the performance of the vacuum tube changes character very markedly. The present analysis is confined strictly to current values less than (or at most, equal to) the limiting value I_m. The space-charge factor ζ is defined in such a way that it varies from a value of zero for no space charge to a value of unity for complete space charge, the latter condition being that in which

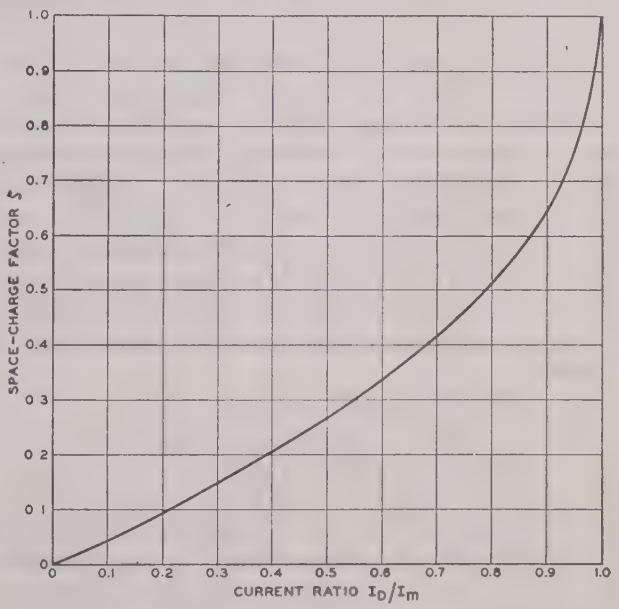

Fig. 2—Graph of space-charge factor ζ.

the injected current has its limiting value I_m. The relation between the actual current I_D, the limiting current I_m, and this space-charge factor ζ may be written

$$I_D/I_m = (9/4)\zeta(1 - \zeta/3)^2. \qquad (3)$$

A graph of this function is shown in Fig. 2, and the formula for computing I_m may be obtained from (3) by setting ζ equal to unity and eliminating T between the second and third equations.[10] It is

$$\frac{\eta}{\epsilon} I_m = \left[\frac{2}{9} \frac{(u_a + u_b)^3}{x^2}\right].$$

In more convenient form, the velocities u_a and u_b may be replaced by the potentials V_{Da} and V_{Db} as given by (1) and we have

$$I_m = \frac{2.33}{10^6} \frac{(\sqrt{V_{Da}} + \sqrt{V_{Db}})^3}{x^2} \qquad (4)$$

which will be recognized by electronics engineers as a

[10] It should be noted that we are concerned only with that solution of ζ which is physically realizable.

somewhat extended form of the familiar Child's equation which applies only to the case where the a plane coincides with a thermionic cathode, the potential V_{Da} then becoming practically zero.

In practice, the value of I_m is calculated from (4). Then given the actual current I_D, the space-charge factor ζ is immediately obtained from the graph of Fig. 2. On first sight, the introduction of the space-charge factor may seem a useless encumbrance, but its utility has been amply justified.

The alternating-current equations are now presented in terms of the space-charge factor rather than in terms of the direct current with the result that the different terms in the equations appear in such a form that their relative magnitudes may be directly compared. There are a great many possible choices for the space-charge factor, but after several trials, the form now presented appears to be the simplest, and it is easy to remember that its value always lies between the two limits of zero for no space charge and unity for complete space charge. Moreover, between two grids of a vacuum tube, its value is often extremely small and alternating-current terms containing it as a factor may then be disregarded with complete confidence, whereas in the older form of the alternating-current equations it was by no means a simple matter to decide when to disregard certain terms.

With this introduction, we are ready for the alternating-current electronics equations and present them as follows:[11]

$$\left.\begin{aligned} V_b - V_a &= A^*I + B^*q_a + C^*v_a \\ q_b &= D^*I + E^*q_a + F^*v_a \\ v_b &= G^*I + H^*q_a + I^*v_a \end{aligned}\right\}. \qquad (5)$$

The coefficients A^* through I^* are expressible in terms of direct-current quantities already defined together with the frequency of the alternating current considered. In doing this, it is convenient to use the usual complex current notation employed in alternating-current theory so that the coefficient A^* for example has the form and properties of a circuit impedance with both real and imaginary components. A symbol θ will also be used to represent the transit angle which is defined by the relations

$$\theta = \omega T \quad \text{and} \quad \beta = i\theta, \quad i = \sqrt{-1}, \qquad (6)$$

ω being the angular frequency as usual and T being the direct-current electron transit time between the a plane and the b plane. Three other special symbols, P, Q, and S will be employed for conciseness in writing the coefficients. They are defined on Table I together with their series expansions in powers of the transit-angle factor β, these latter forms being especially useful at low frequencies where β has a relatively small magnitude.

Besides the formulas for the coefficients in (5), Tables I and II contain a summary of all the formulas presented heretofore, both alternating- and direct-current and also contain the limiting forms for the coefficients when the space charge approaches zero and when it approaches completeness, as found by allowing ζ to take the values 0 and 1, respectively.

[11] It should be noted carefully that the form of (5) is slightly different from (4.15), (4.16) and (4.17) in reference 1.

TABLE I

ELECTRONICS EQUATIONS

Numerics Employed:

$$\eta = 10^7 \frac{e}{m} = 1.77 \times 10^{15}, \qquad \epsilon = 1/(36\pi \times 10^{11}) \frac{\eta}{\epsilon} \doteq 2 \times 10^{28}$$

Direct-Current Equations:

Potential-velocity: $\eta V_D = \frac{1}{2}u^2$ (1)

Space-charge-factor definition: $\zeta = 3(1 - T_0/T)$

Distance: $x = (1-\zeta/3)(u_a+u_b)T/2$ (2)

Current density: $(\eta/\epsilon)I_D = (u_a+u_b)2\zeta/T^2$

Space-charge ratio: $I_D/I_m = (9/4)\zeta(1-\zeta/3)^2$ (3)

(See Fig. 2 for graph)

Limiting-current density:

$$I_m = \frac{2.33}{10^6} \frac{(\sqrt{V_{Da}} + \sqrt{V_{Db}})^3}{x^2} \qquad (4)$$

Alternating-Current Equations:

Symbols employed:

$$\beta = i\theta, \qquad \theta = \omega T, \qquad i = \sqrt{-1}$$

$$P = 1 - e^{-\beta} - \beta e^{-\beta} \doteq \frac{\beta^2}{2} - \frac{\beta^3}{3} + \frac{\beta^4}{8} \cdots$$

$$Q = 1 - e^{-\beta} \doteq \beta - \frac{\beta^2}{2} + \frac{\beta^3}{6} - \frac{\beta^4}{24} \cdots$$

$$S = 2 - 2e^{-\beta} - \beta - \beta e^{-\beta} \doteq -\frac{\beta^3}{6} + \frac{\beta^4}{12} - \frac{\beta^5}{40} + \frac{\beta^6}{180} \cdots$$

General equations for alternating current

q = alternating conduction-current density

v = alternating velocity

$$\left.\begin{aligned} V_b - V_a &= A^*I + B^*q_a + C^*v_a \\ q_b &= D^*I + E^*q_a + F^*v_a \\ v_b &= G^*I + H^*q_a + I^*v_a \end{aligned}\right\} \qquad (5)$$

It may appear at first sight that the mass of equations contained in Tables I and II is of such complexity that it will be practically impossible to unravel a result of any practical application. Such is not the case however, and a few examples will serve to show how quickly the applications may be interpreted in several simple cases. Of course, other more complicated applications lead to more complexity in the analytical formulation, but even there, the interpretation is usually readily apparent, and quantitative calculations may be made with a reasonable amount of labor.

PROPERTIES OF DIODES

As a first example consider the case where no electrons at all are present between the a plane and the b plane in Fig. 1. Evidently then the injected alternating current q_a is zero, and likewise the space charge is zero,

TABLE II

VALUES OF ALTERNATING-CURRENT COEFFICIENTS

$$A^* = \frac{1}{\epsilon}(u_a + u_b)\frac{T^2}{2}\frac{1}{\beta}\left[1 - \frac{\zeta}{3}\left(1 - \frac{12S}{\beta^3}\right)\right]$$

$$B^* = \frac{1}{\epsilon}\frac{T^2}{\beta^3}\left[u_a(P - \beta Q) - u_b P + \zeta(u_a + u_b)P\right]$$

$$C^* = -\frac{1}{\eta}2\zeta(u_a + u_b)\frac{P}{\beta^2}$$

$$D^* = 2\zeta\left(\frac{u_a + u_b}{u_b}\right)\frac{P}{\beta^2}$$

$$E^* = \frac{1}{u_b}\left[u_b - \zeta(u_a + u_b)\right]e^{-\beta}$$

$$F^* = \frac{\epsilon}{\eta}\frac{2\zeta}{T^2}\left(\frac{u_a + u_b}{u_b}\right)\beta e^{-\beta}$$

$$G^* = -\frac{\eta}{\epsilon}\frac{T^2}{\beta^3}\frac{1}{u_b}\left[u_b(P - \beta Q) - u_a P + \zeta(u_a + u_b)P\right]$$

$$H^* = -\frac{\eta}{\epsilon}\frac{T^2}{2}\left(\frac{u_a + u_b}{u_b}\right)(1 - \zeta)\frac{e^{-\beta}}{\beta}$$

$$I^* = \frac{1}{u_b}\left[u_a - \zeta(u_a + u_b)\right]e^{-\beta}$$

Complete space-charge, $\zeta = 1$.

$$A^* = \frac{1}{\epsilon}(u_a + u_b)\frac{T^2}{3\beta}\left(1 + \frac{6S}{\beta^3}\right)$$

$$B^* = \frac{1}{\epsilon}\frac{T^2}{\beta^3}u_a(2P - \beta Q)$$

$$C^* = -\frac{2}{\eta}(u_a + u_b)\frac{P}{\beta^2}$$

$$D^* = 2\left(\frac{u_a + u_b}{u_b}\right)\frac{P}{\beta^2}$$

$$E^* = -\frac{u_a}{u_b}e^{-\beta}$$

$$F^* = \frac{\epsilon}{\eta}\frac{2}{T^2}\left(\frac{u_a + u_b}{u_b}\right)\beta e^{-\beta}$$

$$G^* = -\frac{\eta}{\epsilon}\frac{T^2}{\beta^3}(2P - \beta Q)$$

$$H^* = 0$$

$$I^* = -e^{-\beta}$$

Zero space-charge, $\zeta = 0$.

$$A^* = \frac{1}{\epsilon}(u_a + u_b)\frac{T^2}{2}\frac{1}{\beta}$$

$$B^* = \frac{1}{\epsilon}\frac{T^2}{\beta^3}\left[u_a(P - \beta Q) - u_b P\right]$$

$$C^* = 0$$

$$D^* = 0$$

$$E^* = e^{-\beta}$$

$$F^* = 0$$

$$G^* = -\frac{\eta}{\epsilon}\frac{T^2}{\beta^3}\frac{1}{u_b}\left[u_b(P - \beta Q) - u_a P\right]$$

$$H^* = -\frac{\eta}{\epsilon}\frac{T^2}{2}\left(\frac{u_a + u_b}{u_b}\right)\frac{e^{-\beta}}{\beta}$$

$$I^* = \frac{u_a}{u_b}e^{-\beta}$$

giving $\zeta = 0$. With this value, the coefficient C^* on Table II is zero, and the first of equations (5) on Table I becomes

$$V_b - V_a = A^* I. \tag{7}$$

This shows immediately that A^* is the impedance between unit area of the two parallels planes. From Table II, for zero space charge, the formula for A^* is $A^* = (1/\epsilon)(u_a + u_b)(T_0^2/2)(1/\beta)$. However, from (2) for $\zeta = 0$ we have $x = (u_a + u_b)(T_0/2)$. Substituting this into the above expression for A^* and remembering that $\beta = i\omega T$ we have

$$A^* = x/\epsilon i\omega. \tag{8}$$

Now (8) is precisely the equation for the impedance between two parallel plane conductors in vacuum, and may be written

$$A^* = 1/i\omega C, \qquad C = \epsilon/x \tag{9}$$

which shows, in agreement with well-known relations, that the capacitance per unit area between parallel planes is ϵ/x farads.

As a second example, the same pair of parallel planes will be considered, but it will now be postulated that one of the planes is a thermionic electron-emitting cathode. If this is taken to be the a plane, and if the b plane is operated at a positive direct-current potential with respect to the a plane, then electrons move across the intervening space in accordance with Fig. 1. There are two cases to be considered. In the first case, the direct-current potential on the b plane is sufficiently high to draw off all of the emission from the cathode so that the initial alternating conduction current q_a is necessarily zero. In the second case the direct-current potential is low enough so that only part of the available cathode emission is drawn off, which happens when the electric field intensity at the surface of the cathode approaches zero, and corresponds to the complete space-charge condition where $\zeta = 1$. Thus, on Table II for complete space charge, and for the initial velocity u_a equal to zero, the coefficient B^* likewise becomes zero. It follows then, that in any event, the $B^* q_a$ term of the first of equations (5) on Table I is zero for parallel planes where one of them is a thermionic cathode, regardless of the degree of space charge.

Moreover, the initial electron velocity is determined solely by the cathode temperature, and is not affected

by the alternating current, so that v_a is zero as far as the signal is concerned.

For a diode vacuum tube, therefore, the complete electronics alternating-current equation is, from (5)

$$V_b - V_a = A^*I \tag{10}$$

and differs from (7) only in the fact that ζ is not put equal to zero in writing the expression for A^* from Table II.

The generalization can be extended even further. Whenever electrons are injected across the a plane of a parallel-plane arrangement in a constant stream so that q_a and v_a are zero, the diode equation is again given by (10). Thus the coefficient A^* is the general impedance of a diode whether one of the planes be a thermionic cathode, with or without complete space charge or whether electrons are injected across the initial plane in a constant stream.

It will be helpful to investigate the characteristics of this general diode impedance in greater detail. As a foundation for the diode impedance characteristics the most useful basis is the complete space-charge condition. The equations needed are the expression for A^* taken from Table II with $\zeta=1$ and the expressions for direct-current potential, V_D, and direct-current density I_D from (2) on Table I. Thus we have for complete space charge

$$\left.\begin{aligned} z = A^*_{\zeta=1} &= (1/\epsilon)(u_a+u_b)(T^2/3)(1/\beta)\left[1+6S/\beta^3\right] \\ \eta(\sqrt{V_{Da}}+\sqrt{V_{Db}}) &= \tfrac{1}{2}(u_a+u_b) \\ (\eta/\epsilon)I_D &= (u_a+u_b)2/T^2 \end{aligned}\right\}. \tag{11}$$

By combination of these three equations, the transit time T and the direct-current velocities u_a and u_b may be replaced by the direct current I_D and the direct-current potential V_{Da} and V_{Db} giving

$$z = \frac{2}{3}\,\frac{(\sqrt{V_{Da}}+\sqrt{V_{Db}})^2}{I_D}\left[\frac{2}{\beta}+\frac{12S}{\beta^4}\right]. \tag{12}$$

The coefficient $2(\sqrt{V_{Da}}+\sqrt{V_{Db}})^2/3I_D$ in this expression is of special significance. When the a plane is a thermionic cathode, so that V_{Da} may be taken as zero, corresponding to zero electron velocity of emission, the coefficient is merely $2V_{Db}/3I_D$. This is the expression for the inverse slope of the static characteristic of a diode operating with complete space charge, and hence may be represented by the symbol r_0, the zero-frequency value of the diode resistance.

In the more general case of (11) where the electron velocities at the a plane are not necessarily zero, but where the coefficient has the generalized form $2(\sqrt{V_{Da}}+\sqrt{V_{Db}})^2/3I_D$ we may still denote it by r_0, because for low frequencies, the bracketed factor in (12) reduces to unity, as may be proved by using the series expansion for S given in Table I, and allowing β to approach zero.

In general, then, for a diode with complete space-charge we have

$$z = r_0\left[\frac{2}{\beta}+\frac{12S}{\beta^4}\right] \tag{13}$$

where $r_0=2(\sqrt{V_{Da}}+\sqrt{V_{Db}})^2/3I_D$.

For $z=r+ix$, Table III contains the calculated values for the diode resistance r and reactance x as a function of frequency in terms of the transit angle θ given by $\beta=i\theta=i\omega T$ and Fig. 3 shows a graph of the same data. Also on Fig. 3 the dotted curve represents the reactance of a parallel-plate condenser with the same dimensions as the space-charge diode. It is related to the transit angle as follows:

At extremely high frequencies, the impedance (13) approaches the value $z_\infty \doteq r_0(2/\beta)$ or, by use of (11) instead of (12) the same impedance may be written $z_\infty \doteq (1/\epsilon)(u_a+u_b)(T^2/6)(2/\beta)$. Comparison shows that $r_0=(1/\epsilon)(u_a+u_b)(T^2/6)$. However, from Table I for complete space charge $x=(u_a+u_b)T/3$ so that r_0 may be

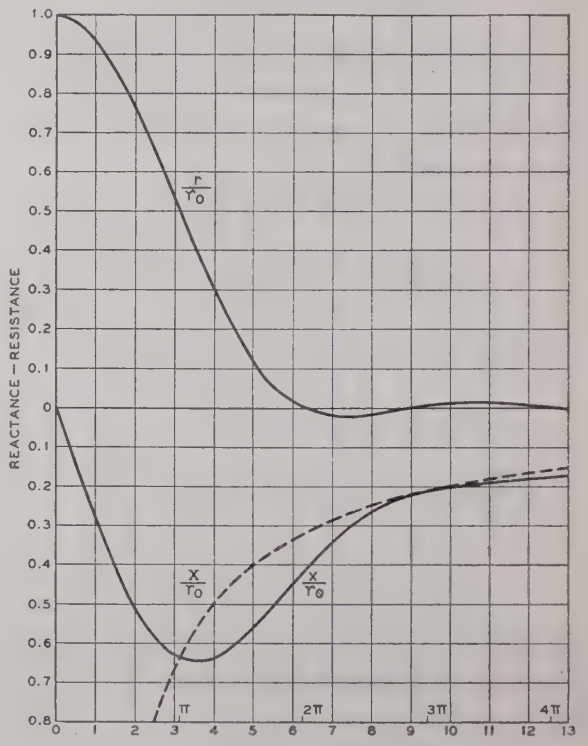

Fig. 3—Diode impedance with complete space charge.

written $r_0=(x/\epsilon)(T/2)=(x/\epsilon i\omega)(\beta/2)$. From the explanation given in connection with (8) it is evident that $x/\epsilon i\omega$ is the impedance of a parallel-plate condenser coinciding in dimensions with the diode now under consideration. The diode reactance for extremely high frequencies is therefore identical with that of a parallel-plate condenser in free space, and is not modified by the presence of electrons. The dotted curve in Fig. 3 is obtained from the relation

$$X_c = r_0(2/\theta) \tag{14}$$

which may be written immediately from the foregoing discussion. In the form $T=2Cr_0$ this equation has been discussed at some length by Benham.[3]

At lower frequencies, the space-charge diode reactance departs from this value and eventually approaches zero. It is often convenient, however, to work with

admittances and Fig. 4 is a graph of the data used in Fig. 3, but plotted on an admittance basis. Also, on Fig. 4 there are plotted the low- as well as the high-frequency asymptotes of the susceptance. The former is given by $(3\theta/10)g_0=(3/5)\omega\epsilon/x$ and the latter by $(\theta/2)g_0=\omega\epsilon/x$, where $g_0=1/r_0$.

The low-frequency impedance consequently may be represented as a resistance in parallel with a capacitance. More elaborate circuits to cover a wider frequency range may be devised. One such circuit has been discussed by Benham.[3] Rather than try to find a generally valid equivalent diode network which would be quite

TABLE III
DIODE RESISTANCE AND REACTANCE

θ	r/r_0	$-x/r_0$
0	1	0.
0.4	0.9894	0.1192
1.0	0.9351	0.2883
1.4	0.8760	0.3886
$\frac{\pi}{2}=1.571$	0.8460	0.4272
1.8	0.8019	0.4748
$\frac{3}{4}\pi=2.356$	0.6806	0.5669
2.8	0.5752	0.6151
$\pi=3.142$	0.4928	0.6366
3.6	0.3848	0.6453
4.0	0.2969	0.6359
$\frac{3}{2}\pi=4.712$	0.1633	0.5878
5.2	0.0928	0.5388
5.6	0.0486	0.4939
$2\pi=6.283$	0.	0.4151
6.8	−0.0174	0.3599
7.2	−0.0220	0.3224
$\frac{5}{2}\pi=7.85$	−0.0185	0.2731
8.4	−0.00998	0.2437
9.0	−0.00021	0.2222
$3\pi=9.42$	+0.006084	0.2122
10.	+0.01094	0.2032
$\frac{7}{2}\pi=10.99$	+0.010669	0.1926
$4\pi=12.56$	0.	0.1712

complicated, it is thought better to consider the diode complete in itself as a new impedance element and to become familiar with its characteristics.

The resistive component of the diode impedance has been discussed many times, but it is important to note again that it passes through zero at transit angles of 2π, 4π, and so forth, and again at transit angles of approximately 3π, 5π, and so forth, being negative in sign whenever the transit angle lies between any whole number of cycles and that number increased by approximately a half cycle. The values of these negative maxima are given to a close approximation by

$$r_- = -12r_0/\theta^3$$
$$\theta = 2\pi n + \pi/2 = (\pi/2)(1+4n) \quad n = 1, 2, 3, \cdots . \quad (15)$$

At this point, it may be well to remind the reader of the fact that the electronics equations are based on an idealization which assumes that all electrons start from the cathode with the same (extremely small) velocity and that none of them turns back. Actually, the velocities of emission follow the Maxwellian distribution, and with normal voltages on the anode, a potential minimum or space-charge barrier is formed in front of the cathode, before which large numbers of electrons stop and then return to the cathode. The electronics idealization becomes more exact when the a plane in Fig. 1 is taken just beyond this potential minimum, so that the region under analysis contains no returning electrons. When this is done, the impedance given by the equations and shown graphically in Figs. 3 and 4 must be modified by the addition of the impedance between the a plane and the actual cathode in order to give the impedance presented between the actual electrodes of the tube. It can be shown that the impedance between the a plane and cathode contains a resistive component which becomes quite important at high frequencies and for small current densities so that the actual negative resistance exhibited in Fig. 3 may be decreased or even masked completely unless care is taken to reduce the cathode temperature to a point where the number of emitted electrons is not greatly in excess of that required to furnish those flowing to the anode.

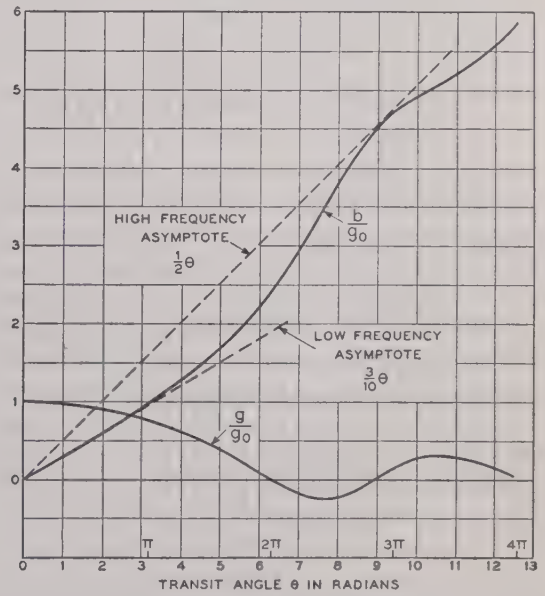

Fig. 4—Diode admittance with complete space charge.

We have now seen how to apply the coefficient A^* to find the impedance of planar diodes with no space charge and with complete space charge. The remaining case of partial space charge in diodes is managed in the same way, and no particular difficulty will be experienced in handling the resistive component of the impedance; in fact it is given precisely by the graph of Fig. 3, and the factor r_0 requires only a minor modification from the form given under (13). The reactive component is more complicated, and the difference is made apparent when a thermionic cathode is taken as the a plane. For complete space charge, the static characteristic of current versus voltage follows the well-known Child's equation, given by (4) with V_{Da} set equal to zero. The slope of that curve gives the zero-frequency resistance r_0, and the zero-frequency reactance approaches zero. However, just as soon as the applied voltage is made high enough to draw off all of the electrons which the cathode is capable of emitting, complete space charge no longer exists, and the voltage-current characteristic has

a slope indicative of infinite resistance at zero frequency. It follows that the zero-frequency impedance of the diode with incomplete space charge is reactive, and that the reactance approaches infinity just as in a condenser, as the frequency approaches zero. Thus the reactance curve of Fig. 3 cannot apply to partial space charge, though as we shall see, the dotted curve for zero space charge does not apply exactly either.

In this general case, the resistive and reactive components of the impedance A^* are conveniently separated by writing $i\theta$ for β and separating the real and the imaginary parts. By using the direct-current relations on Table I to eliminate T and after some minor algebraic rearrangement, we have

$$A^* = \frac{2}{3}\zeta^2 \frac{(\sqrt{V_{Da}}+\sqrt{V_{Db}})^2}{I_D}\frac{12}{\theta^4}[2(1-\cos\theta)-\theta\sin\theta] - i\frac{2}{3}\zeta^2\frac{(\sqrt{V_{Da}}+\sqrt{V_{Db}})^2}{I_D}\frac{12}{\theta^4}\cdot\left[\frac{\theta^3}{6}+\theta(1+\cos\theta)-2\sin\theta\right] - i\frac{x}{\omega\epsilon}\frac{(1-\zeta)}{(1-\zeta/3)}. \quad (16)$$

Here the first term on the right is the resistive component, while the last two together constitute the reactive component. For complete space charge, this should reduce to the form (13), which it does when ζ is made unity. Now ζ appears merely as a multiplier in the first two terms on the right and consequently Table III and Fig. 3 represent the general resistance characteristic of parallel-plane diode where the zero-frequency series resistance r_0 is redefined as

$$r_0 = \frac{2}{3}\zeta^2\frac{(\sqrt{V_{Da}}+\sqrt{V_{Db}})^2}{I_D}. \quad (17)$$

They also represent one component of the general diode reactance, with (17) for r_0. But this reactance appears in series with a simple capacitance, whose reactance is given by the last term in (16). The capacitance is $(\epsilon/x)(1-\zeta/3)/(1-\zeta)$, which approaches infinity as ζ approaches unity.

The general diode impedance between virtual cathode and anode thus consists of three impedance elements in series. The first is a resistive impedance given by the resistance curve of Fig. 3. The second is a reactive impedance given by the reactance curve of Fig. 3. The third is a simple capacitance whose value is $C(1-\zeta/3)/(1-\zeta)$ where C, as in (9), is the capacitance between parallel conductors in free space, separated by the distance between the a plane and the b plane of the actual diode under consideration.

At very high frequencies, the general impedance (16) approaches

$$A^* \doteq -12r_0(\sin\theta)/\theta^3 - i(x/\omega\epsilon) \quad (18)$$

as may be seen by expressing the coefficient of the second term on the right of (16) in terms of x and then combining the second with the last term, letting θ become very large and discarding lower powers of θ in comparison with higher powers. This is a resistance in series with a simple capacitance, and the impedance of both is again given by Fig. 3 for large values of θ. The capacitance again is merely that which would exist between parallel conducting plates located at the a plane and the b plane and having only free space between them.

Oscillation possibilities of diodes are indicated by (18) but they have been discussed before[4,12] and the discussion will not be repeated here.

Multielement Tubes in General

So much then for diodes. The building up of the equivalent network of the multielement vacuum tube may now be undertaken. The picture is illustrated by Fig. 2 and the method is merely to imagine several cascade arrangements of the parallel plane diode geometry of Fig. 1. In Fig. 5, a tetrode is shown for example, and between the cathode and control grid there exist conditions analogous to those shown in Fig. 1 when the proper values for the boundary conditions at a and b are selected. Again, between the control grid and the screen another parallel plane diode may be envisioned with different boundary conditions and different values of the space-charge factor from those existing in the first-named region. A similar diode is located between the screen and the plate. The joining together of the cascaded-diode arrangement is accomplished by an approximation which experience and analysis have shown to be very nearly exact. The grid wires themselves disturb the simple uniform relations of a parallel-plane-diode arrangement. We, therefore, imagine the fictitious planes separating consecutive diodes to be located extremely near the grid wires but not quite including them. However, the final plane for the region (1) in Fig. 5 and

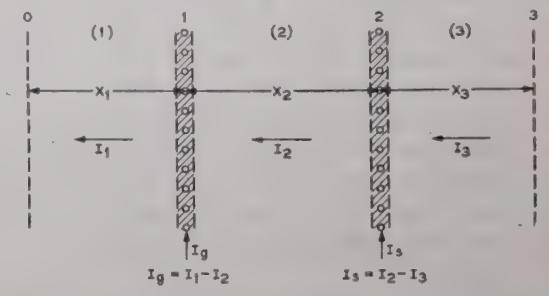

Fig. 5—General diagram of multielement vacuum tube.

the initial plane for region (2) are taken to be so close together that their potentials, both alternating and direct, are the same. This potential is called the "effective potential" of the grid. Its value is determined in such a way that currents and potentials existing in the consecutive diodes are identical with those which would occur if the grid were removed and substituted by a solid

[12] F. B. Llewellyn and A. E. Bowen, "Ultra-high-frequency oscillations by means of diodes," *Bell Sys. Tech. Jour.*, vol. 18, pp. 280–291; April, 1939.

metallic plate having on its two surfaces the requisite boundary conditions; that is, of conduction current and electron velocity entering the one surface and leaving the other. The grid wires themselves are at a different potential from this effective potential of the grid plane and the grid current is the difference between the total current flowing out of the left-hand surface of the fictitious plane and that flowing into its right-hand surface, illustrated by the difference between I_1 and I_2 in Fig. 5. In order to provide for this difference in potential between the grid wires and the grid plane, it is obviously necessary to suppose the proper impedances and current sources to be connected between the grid wires and the fictitious solid plane at the effective potential of the grid. These questions will be dealt with later.

For the present, attention is confined to the main electron stream which originates at plane (0) in Fig. 5, either from a hot cathode or by being injected through the plane at a constant rate and velocity. In region (1) between planes 0 and 1, therefore, the simple diode equations apply directly, and the impedance is given by A^* in (10) or (in greater detail) in (16).

For the region (2) in Fig. 5 conditions are not quite so simple, because the electrons do not cross plane 1 and enter region (2) in a smooth continuous stream, but on the contrary, they enter in groups or bunches moving at variable velocities, having been acted on by the high-frequency voltage between 0 and 1. However, (5) on Table I provides the means of calculating the initial conduction current and velocity of electrons injected into region (2) because the electrons enter region (2) with the same velocity with which they leave region (1), and the conduction current entering region (2) must also be the same as that leaving region (1) whenever the grid at 1 is at a negative potential, so that no electrons strike it and are thus prevented from moving into region (2). When this is not the case (that is, when the grid is positive, and therefore the wires collect some of the approaching electrons), the conduction current per square centimeter injected into region (2) is less than that leaving region (1). The fraction α may be used to represent this decrease in conduction current so that, if q is the conduction current leaving region (1), then αq is the conduction current entering region (2). The fraction $(1-\alpha)$ is the differential capture fraction of the grid; that is, it is the ratio of the increment of electron current captured by the grid to a small increment of emitted electron current.

Denoting conditions at the right-hand boundary of region (1) by the subscript 1, we have then, from Table I, for $V_a=0$: $V_1=A_1^*I_1, q_1=D_1^*I_1+E_1^*q_0, v_1=G_1^*I_1$ where the remaining terms of (5) have disappeared because of the initial conditions specified for region (1). Moreover, when plane (0) is a thermionic cathode with complete space charge, E_1^* is zero because u_a is then zero on Table II. When there is not complete space charge q_0 is zero, so that the $E_1^*q_0$ term above may be dropped in either case. It will be found convenient in later work to express these relations in terms of V_1, in which case they become:

$$I_1 = V_1/A_1^*, \quad q_1 = V_1(D_1^*/A_1^*), \quad v_1 = V_1(G_1^*/A_1^*). \tag{19}$$

For region (2) and denoting conditions at the right-hand end of region (2) by the subscript 2, we have from (5) on Table I

$$\begin{aligned} V_2 - V_1 &= A_2^*I_2 + B_2^*\alpha_1 q_1 + C_2^* v_1 \\ q_2 &= D_2^*I_2 + E_2^*\alpha_1 q_1 + F_2^* v_1 \\ v_2 &= G_2^*I_2 + H_2^*\alpha_1 q_1 + I_2^* v_1. \end{aligned}$$

From (19) the q_1 and v_1 may be eliminated giving

$$\left.\begin{aligned} I_2 &= (V_2 - V_1)/A_2^* - (V_1/A_1^*A_2^*)(B_2^*\alpha_1 D_1^* + C_2^* G_1^*) \\ q_2 &= (V_2 - V_1)D_2^*/A_2^* + (V_1/A_1^*A_2^*)[A_2^*(D_1^*\alpha_1 E_2^* + G_1^* F_2^*) - D_2^*(D_1^*\alpha_1 B_2^* + G_1^* C_2^*)] \\ v_2 &= (V_2 - V_1)G_2^*/A_2^* + (V_1/A_1^*A_2^*)[A_2^*(D_1^*\alpha_1 H_2^* + G_1^* I_2^*) - G_2^*(D_1^*\alpha_1 B_2^* + G_1^* C_2^*)] \end{aligned}\right\}. \tag{20}$$

For regions (3), (4), etc., a similar procedure is followed and the results may be summarized by writing

$$\left.\begin{aligned} I_1 &= V_1 y_{11} \\ I_2 &= (V_2 - V_1)y_{22} - V_1 y_{12} \\ I_3 &= (V_3 - V_2)y_{33} - (V_2 - V_1)y_{23} - V_1 y_{13} \\ I_4 &= (V_4 - V_3)y_{44} - (V_3 - V_2)y_{34} \\ &\quad - (V_2 - V_1)y_{24} - V_1 y_{14} \end{aligned}\right\}, \tag{21}$$

etc., where the admittances are given by

$$\left.\begin{aligned} y_{11} &= 1/A_1^*, \quad y_{22} = 1/A_2^*, \quad y_{33} = 1/A_3^*, \text{ etc.} \\ y_{12} &= (1/A_1^*A_2^*)(D_1^*\alpha_1 B_2^* + G_1^* C_2^*) \\ y_{23} &= (1/A_2^*A_3^*)(D_2^*\alpha_2 B_3^* + G_2^* C_3^*) \\ y_{13} &= (1/A_1^*A_2^*A_3^*)[A_2^*\{\alpha_2 B_3^*(D_1^*\alpha_1 E_2^* + G_1^* F_2^*) \\ &\quad + C_3^*(D_1^*\alpha_1 H_2^* + G_1^* I_2^*)\} \\ &\quad - \{\alpha_2 B_3^* D_2^*(D_1^*\alpha_1 B_2^* + G_1^* C_2^*) \\ &\quad + C_3^* G_2^*(D_1^*\alpha_1 B_2^* + G_1^* C_2^*)\}] \end{aligned}\right\}, \tag{22}$$

etc.

The formulation of (21) immediately suggests that any region, say the third, can be represented as shown in Fig. 6. Here the third region is represented by a box with the current I_3 flowing into and out of it. The admittance y_{33} has two constant-current (or current-regulated) sources connected across it, one for each preceding region in Fig. 5. One current source impresses the current $I_b=(V_2-V_1)\ y_{23}$ on the admittance y_{33} while the other current source impresses on it the current $I_c=V_1\ y_{13}$. The sum of the currents entering the node at V_3 thus gives $I_3=I_a-I_b-I_c$ which is in accord with (21) and demonstrates the correctness of the equivalent diagram of Fig. 6.

The equivalent diagram of the entire electron stream of Fig. 5 is then as shown in Fig. 7. The constant-current

generators play a role analogous to that of the constant-voltage μ generators with which the older conventional vacuum-tube network represents the control of the plate current by means of the grid voltage. In Fig. 7 the controls on the various regions are in terms of impressed currents rather than impressed voltages and the currents in turn are expressed in terms of the voltages on the equivalent planes of the various grids rather than in terms of the voltages on the grid wires themselves. As soon as a relation between the grid voltage and the voltage on the equivalent grid plane is found, then the admittances y_{12}, y_{13}, etc., may be multiplied by the corresponding factor to give the transadmittance from the

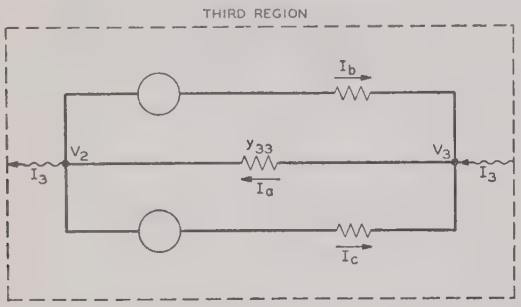

Fig. 6—Equivalent network of third region.

control grid to some other electrode. At low frequencies, these transadmittances should degenerate into our usual transconductances, and it will be shown later that they do just that. First however, it seems advisable to give a more detailed analysis of the transadmittances as applied to the electron stream itself as in Fig. 7.

Discharge Path with Space-Charge Control

For this example a tube operating with complete space charge in region (1) will be taken where the 0 plane represents a thermionic cathode. In succeeding regions, a very good approximation to conditions in the usual type of space-charge control tube will be obtained if we assume the space charge in those regions to be very

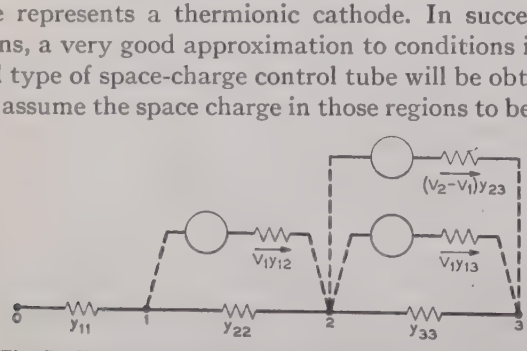

Fig. 7—Equivalent network of electron stream of Fig. 5.

small. Justification for making this approximation may be had by calculating the value of the space-charge factor ζ for most of our conventional triodes, tetrodes, and pentodes. This may be done with the aid of the formulas on Table I together with the graph of Fig. 2.

For example, from (4) on Table I with $V_{Db}=100$, $V_{Da}=4$, $x=0.1$ we require about 400 mils of current per square centimeter to form complete space charge between the grid and the screen. The current actually drawn by such a tube would be in the neighborhood of some 30 milliamperes per square centimeter. From this, $I_D/I_m=0.075$ and thence from Fig. 2 the space-charge factor ζ is only 0.033. In many practical tubes the factor is much smaller than this.

Accordingly, in all but region (1) it will produce a negligible error if we disregard terms containing ζ in the expressions for the coefficients in (22). From Table II it is seen that C^*, D^*, and F^* are then zero for all except region (1). Moreover, in region (1) where we have complete space charge, H^* is zero immediately, and both B^* and E^* are small enough to be neglected because u_a, the electron velocity at the cathode, is only a small fraction of an equivalent volt. The result is that (22) takes the following form:

$$\left.\begin{aligned} &y_{11}=1/A_1^*, \qquad y_{22}=1/A_2^*, \qquad y_{33}=1/A_3^* \\ &y_{12}=\alpha_1 D_1^* B_2^*/A_1^* A_2^* \\ &y_{23}=0 \\ &y_{13}=\alpha_1\alpha_2 D_1^* E_2^* B_3^*/A_1^* A_3^* \end{aligned}\right\}. \qquad (23)$$

The coefficients y_{22} and y_{33} are the reciprocals of the A^* values with no space charge and are shown by (9) to be equal to $i\omega C$, where C is the free-space capacitance between solid planes coinciding with the grids in question. The coefficient y_{11} has likewise been discussed in analyzing complete space-charge diodes, and is plotted on Fig. 4 from data on Table III obtained from (11), or from (16) with $\zeta=1$.

The transadmittances y_{12} and y_{13} in (23) are even more interesting. The fact that the factor $1/A_1^*$ appears in both, shows that they are proportional to $1/r_0$ as given by (13). Now, the reciprocal of r_0 is $(3/2)I_D/V_{D1}$ for region (1) and may conveniently be written g_0. In this form, we recognize $(-g_0)$ as the low-frequency transconductance of the tube referred to the effective potential of the grid. It differs slightly from the cathode mutual conductance g_m by a factor which is approximately equal to $\mu/(1+\mu+4/3\cdot x_2/x_1)$ but which will be derived in more exact form presently.

Detailed expressions for the transadmittances are obtained of course by substituting from Table II into (23), giving

$$y_{12}=\alpha_1\frac{(2P_1)}{(\beta_1^2A_1^*)}\frac{(2/\beta_2^2)}{(u_1+u_2)}\left[u_1(P_2-\beta_2Q_2)-u_2P_2\right] \qquad (24)$$

and

$$y_{13}=\alpha_1\alpha_2\frac{(2P_1)}{(\beta_1^2A_1^*)}e^{-\beta_2}\frac{(2/\beta_3^2)}{(u_2+u_3)}\left[u_2(P_3-\beta_3Q_3)-u_3P_3\right]. \qquad (25)$$

These equations may be plotted for particular cases, but more information of a general nature may be obtained by investigating their behavior, first at moderately low frequencies where the series expansions on Table I for P, Q, and S may be used, and second at extremely high frequencies where the magnitude of $\beta=i\theta$ is large compared to unity. A most significant thing to notice in (25), however, is that in any event the transit time through region (2) appears only in the form $e^{-i\theta_2}$. This means that the sole effect of that region upon regions following it is to delay transmission to them.

It is useful to keep in mind the limiting values at very low and very high frequencies which are approached by the factors grouped within the several sets of parentheses in (24) and (25). These limiting values may be tabulated for reference as follows:

	$\left(\frac{2P_1}{\beta_1^2 A_1^*}\right)$	$\frac{2}{\beta_n^2}\left[\frac{u_{n-1}(P_n-\beta_n Q_n)-u_n P_n}{u_n+u_{n-1}}\right]$
$\beta \doteq 0$	g_0	(-1)
$\beta \doteq \infty$	$-g_0 e^{-\beta_1}$	$-\frac{2}{\beta_n}\left(\frac{u_{n-1}-u_n e^{-\beta_n}}{u_{n-1}+u_n}\right)$

In between these limiting forms, the behavior of $(2P_1)/(\beta_1^2 A_1^*)$ is especially important. Its phase varies widely but its magnitude remains within about 30 per cent of the low-frequency magnitude g_0. Fig. 8 shows the phase and magnitude in terms of the low-frequency magnitude g_0. The low as well as the high-frequency asymptotes of the phase are indicated by the dotted lines. As calculated later, the former is given by $-11\,\theta_1/30$ and the latter is evidently $\pi-\theta_1$. The cross-over point a occurs at $\theta_1=30\,\pi/19$.

Going back now to the general admittances (24) and (25) and writing their low-frequency values in detail, we have

$$y_{12} \doteq -\alpha_1 g_0\left[1-i\left(\frac{11}{30}\theta_1+\frac{1}{3}\theta_2\frac{\sqrt{V_{D1}}+2\sqrt{V_{D2}}}{\sqrt{V_{D1}}+\sqrt{V_{D2}}}\right)-\left(\frac{11}{150}\theta_1^2+\frac{11}{90}\theta_1\theta_2\frac{\sqrt{V_{D1}}+2\sqrt{V_{D2}}}{\sqrt{V_{D1}}+\sqrt{V_{D2}}}+\frac{1}{12}\theta_2^2\frac{\sqrt{V_{D1}}+3\sqrt{V_{D2}}}{\sqrt{V_{D1}}+\sqrt{V_{D2}}}\right)+\cdots\right] \quad (26)$$

and

$$y_{13} \doteq -\alpha_1\alpha_2 g_0\left[1-i\left(\frac{11}{30}\theta_1+\theta_2+\frac{1}{3}\theta_3\frac{\sqrt{V_{D2}}+2\sqrt{V_{D3}}}{\sqrt{V_{D2}}+\sqrt{V_{D3}}}\right)-\left(\frac{11}{150}\theta_1^2+\frac{11}{90}\theta_1\theta_3\frac{\sqrt{V_{D2}}+2\sqrt{V_{D3}}}{\sqrt{V_{D2}}+\sqrt{V_{D3}}}+\frac{1}{12}\theta_3^2\frac{\sqrt{V_{D2}}+3\sqrt{V_{D3}}}{\sqrt{V_{D2}}+\sqrt{V_{D3}}}+\frac{11}{30}\theta_1\theta_2+\frac{1}{3}\theta_2\theta_3\frac{\sqrt{V_{D2}}+2\sqrt{V_{D3}}}{\sqrt{V_{D2}}+\sqrt{V_{D3}}}+\frac{1}{2}\theta_2^2\right)-\cdots\right]. \quad (27)$$

The minus sign appears in these equations, as usual, and denotes the current-voltage relations in vacuum tubes which produce a decrease in output voltage when the control-grid voltage is increased. That is, with the current directions assumed in Fig. 7, the transconductance of a normal vacuum tube turns out to be a *negative* number.[13] The phase of the transadmittance referred to the electron stream may be found from (27) for tetrodes.

[13] This is contrary to the usual practice. To bring the analysis into line with standard practice involves a change in assumed current direction in the second and third regions of Fig. 5, and therefore would have complicated the presentation of the viewpoint.

Thus writing $y_{13}=-\alpha_1\alpha_2 g_0 e^{i\phi}$ we have, for small values of the transit angle θ; that is for small enough values of ϕ so that $\tan\phi$ may be replaced by ϕ itself:

$$\phi = -\left(\frac{11}{30}\theta_1+\theta_2+\frac{1}{3}\theta_3\frac{\sqrt{V_{D2}}+2\sqrt{V_{D3}}}{\sqrt{V_{D2}}+\sqrt{V_{D3}}}\right). \quad (28)$$

When the screen and plate are at approximately the same potential, so that V_{D2} and V_{D3} are about equal, the phase of the transadmittance is

$$\phi = -\left(\frac{11}{30}\theta_1+\theta_2+\frac{1}{2}\theta_3\right). \quad (29)$$

In this form, it is easy to visualize the contribution to the total phase of the transadmittance which is produced by the individual transit angles in the three regions of the tetrode.

For a pentode, the transadmittance from region (1) to the output region (4) would have the phase

$$\phi = -\left(\frac{11}{30}\theta_1+\theta_2+\theta_3+\frac{1}{3}\theta_4\frac{\sqrt{V_{D3}}+2\sqrt{V_{D4}}}{\sqrt{V_{D3}}+\sqrt{V_{D4}}}\right). \quad (30)$$

Fig. 8 shows the contribution of region 1 to the total phase angle.

As far as the magnitude of these transadmittances is concerned, it is evident from (26) and (27) that the frequency produces no appreciable effect until the product of two of the individual transit angles becomes appreciable compared with unity. The first effect of electron transit time is thus to cause a rotation in the phase of the transadmittance with practically no change in its magnitude.

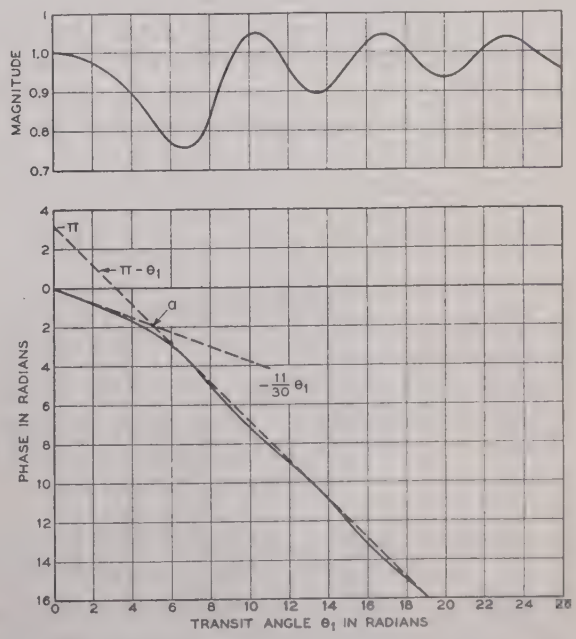

Fig. 8—Phase and magnitude of the transadmittance factor $2P_1/g_0\beta_1^2 A_1^*$.

In (26) and (27) the α factors denote the electrons which are not captured by the various grids in their passage through the tube. In the usual tube where the

first grid is negatively biased, and consequently captures no electrons, the corresponding factor α_1 is unity. If the electrons were focused well enough so that the screen captured no electrons, then α_2 would likewise be unity, but in most practical tubes it lies in the neighborhood of 0.7 or 0.8.

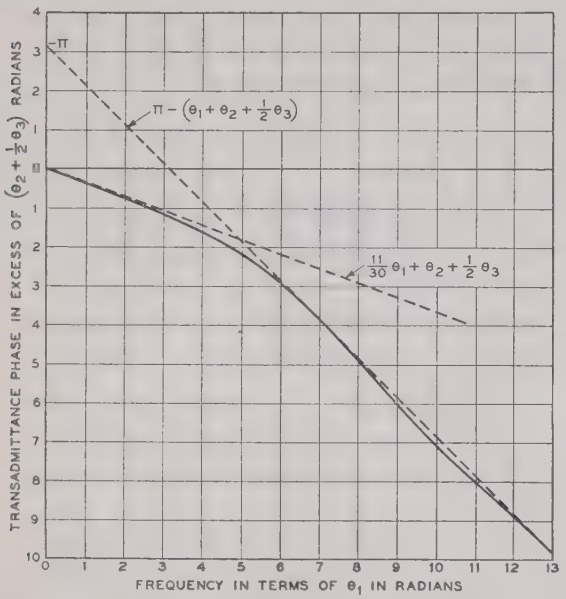

Fig. 9—Phase of transadmittance.

Turning now from these low-frequency approximations to a case where the transit angle in all regions except the last is large compared to unity while in the last region it is very small, we have the following forms for (24) and (25):

$$y_{12} \doteq +g_0\alpha_1 e^{-i\theta_1}\left[1-i\frac{\theta_2}{3}\frac{\sqrt{V_{D1}}+2\sqrt{V_{D2}}}{\sqrt{V_{D1}}+\sqrt{V_{D2}}}\right] \tag{31}$$

$$y_{13} \doteq +g_0\alpha_1\alpha_2 e^{-i(\theta_1+\theta_2)}\left[1-i\frac{\theta_3}{3}\frac{\sqrt{V_{D2}}+2\sqrt{V_{D3}}}{\sqrt{V_{D2}}+\sqrt{V_{D3}}}\right]. \tag{32}$$

Here the effect of all regions preceding the last one is merely to rotate the phase of the transadmittance without changing its magnitude. The final region, which is the second for (31) and the third for (32), does not affect the magnitude so long as the transit angle across it is very small. The effect on phase, however, should be noted, and we have for the tetrode, from (32)

$$\phi = \pi - \left(\theta_1+\theta_2+\frac{1}{3}\theta_3\frac{\sqrt{V_{D2}}+2\sqrt{V_{D3}}}{\sqrt{V_{D2}}+\sqrt{V_{D3}}}\right). \tag{33}$$

This should be compared with (28) and shows that the effect of making the transit angles in regions (1) and (2) very large is merely to change the rate of phase increase with frequency by changing the contribution of region (1) from $11\,\theta_1/30$ to $(\theta-\pi)$. The additive term π enters because the sign preceding the right hand term of (27) is minus while in (32) it is plus, thus indicating a phase shift of 180 degrees. This may be seen by referring to the high-frequency phase asymptote on Fig. 8 which applies to the contribution of θ_1 in triodes and tetrodes.

A sketch of the total phase of the transadmittance of tetrodes is shown in Fig. 9, which is plotted for the case where the effective screen and plate potentials are the same. At low frequencies, the phase follows the straight line labelled $11\,\theta_1/30+\theta_2+\theta_3/2$. At very high frequencies it follows the straight line labelled $\theta_1+\theta_2+\theta_3/2$ which may be extrapolated back to zero frequency and intercepts the ϕ axis at a phase of π radians in accord with (33). At intermediate frequencies the phase of the transadmittance changes from one of these straight lines to the other as shown by the solid line of the drawing. The point where the two straight lines intersect corresponds to $\theta_1=30\pi/19$ as in Fig. 8 regardless of the values of θ_2 and θ_3, and this holds even when the screen and plate effective potentials are not the same, as was assumed in drawing Fig. 9.

When we remove the restriction that the transit angle across the output region is small and replace it by the opposite one, namely that the transit angle across all regions including the input shall be large, then the transadmittances for the triode and tetrode become, from (24) and (25),

$$y_{12} = \alpha_1 g_0 e^{-i\theta_1}\frac{\sqrt{V_{D1}}-\sqrt{V_{D2}}\,e^{-i\theta_2}}{\frac{1}{2}i\theta_2(\sqrt{V_{D1}}+\sqrt{V_{D2}})} \tag{34}$$

$$y_{13} = \alpha_1\alpha_2 g_0 e^{-i(\theta_1+\theta_2)}\frac{\sqrt{V_{D2}}-\sqrt{V_{D3}}\,e^{-i\theta_3}}{\frac{1}{2}i\theta_3(\sqrt{V_{D2}}+\sqrt{V_{D3}})}. \tag{35}$$

These are to be compared with (31) and (32) in investigating the effect of large output transit angles. The relations may be easily seen from (35) when it is assumed that the screen and plate effective potentials are the same so that $V_{D2}=V_{D3}$. When this is the case

$$y_{13} = \alpha_1\alpha_2 g_0\left|\sin\tfrac{1}{2}\theta_3/\tfrac{1}{2}\theta_3\right| e^{-i(\theta_1+\theta_2+\frac{1}{2}\theta_3)}. \tag{36}$$

The phase again follows the high-frequency asymptote of Fig. 9. The magnitude follows a fluctuating curve as shown in Fig. 10, which, however, applies only for the large transit angles assumed in (36). Whenever $\sin\theta_3/2$ has the value of unity, the transadmittance is inversely proportional to $\theta_3/2$, and hence decreases as the frequency increases. This would apparently indicate that the gain of an amplifier tube would necessarily decrease when the output transit angle is made large, but such a sweeping generalization is in fact not warranted, as the following argument will show.

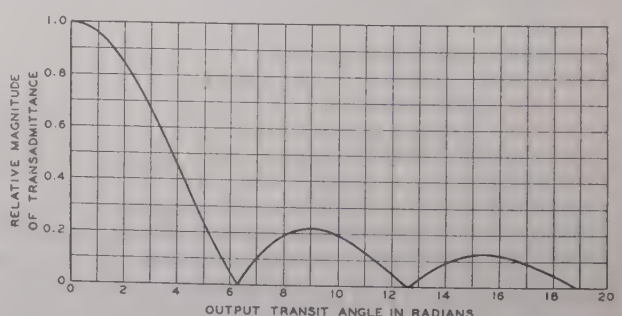

Fig. 10—Magnitude of transadmittance as a function of output transit angle.

The voltage gain of an amplifier, measured in terms of an input voltage, which for this example may be taken as V_1, is given by the expression.

$$G = \alpha_1\alpha_2 g_0 XQ(1/\tfrac{1}{2}\theta_3) \qquad (37)$$

where XQ is the antiresonant impedance of a simple tuned circuit connected across the output. The symbol Q has its usual meaning of the ratio of reactance to resistance of the inductance element, and X is the reactance of either the inductance or the capacitance element of the circuit. Here we take it to be the capacitance, and assume that the output capacitance reactance, $1/\omega C_3 = 1/|y_{33}|$ in Fig. 7 comprises the whole of X. Writing this value into (37) we have $G=\alpha_1\alpha_2 g_0\ Q(1/\tfrac{1}{2}\theta_3|y_{33}|) = g_0\ Q(|A_3^*|/\tfrac{1}{2}\theta_3)$. From Table II, for no space charge the value of A_3^* may be inserted in terms of the velocities, u_2 and u_3 which determine the potentials V_{D2} and V_{D3}. This gives

$$G = (\alpha_1\alpha_2 g_0 Q/\omega^2\epsilon)(u_2 + u_3) \qquad (38)$$

and shows that as long as the potentials on screen and plate are held fixed so that u_2 and u_3 are constant, the gain does not change at a given frequency when the output space between planes 2 and 3 is lengthened. In the lengthening process, the capacitance naturally decreases so that a corresponding increase in tuning inductance is required, and for constancy of gain this must be accomplished without changing its Q.

When the effect of change in frequency is considered, the formulas tell another story. The gain is inversely proportional to the square of the frequency under the conditions of (38), for constant Q and constant biasing voltages, and when the output region is adjusted to maintain the transit angle such that $\sin\theta_3/2$ is unity. This does not look particularly favorable for very high frequencies and the difficulty arises from the restrictions of voltage and transit angle placed upon (38). A better idea of what may be accomplished at high frequencies may be obtained by supposing that V_{D2} and V_{D3} are high enough and that the distance x_3 is short enough so that the transit angle across the output region is very small. In that event (32) expresses the transadmittance and the gain may be written

$$G = \alpha_1\alpha_2 g_0 XQ \qquad (39)$$

which differs from (37) by the absence of $\theta_3/2$ in the denominator. Replacing X by its equivalent in terms of the distance x_3 we have

$$G = \alpha_1\alpha_2 g_0 Q x_3/\omega\epsilon \qquad (40)$$

which now replaces (38). Hence, when the output transit angle is small, the gain is independent of frequency if the value of the output capacitive reactance is held constant, for a given Q. This is merely the situation which has been encountered from the time when the first vacuum-tube amplifier was built, and is familiar to all amplifier-circuit engineers. The only new features introduced here by ultra-high-frequency operation are the restriction that the output transit angle shall be small, and the property of the amplifier that the output phase may lag considerably behind that of the input as shown in the discussion in connection with (35) and Fig. 9.

These gains expressed by the preceding equations are in terms of voltage only and have to be interpreted in terms of power before they become really significant. In doing this, the input impedance to the vacuum tube becomes a controlling factor, and the remarks made just following (15) take on added importance.

Extension to External Terminals

The foregoing analysis applies to gain in voltage from a given high-frequency voltage V_1 applied between the equivalent plane of the control grid and the cathode and to an output measured between the equivalent plane of the screen (or suppressor in pentodes) and the plate. It says nothing about the loss in getting from a signal on the grid wires to the voltage on the equivalent grid plane, nor does it include a drop in output occasioned by getting from the equivalent plane of the screen out to the screen wires themselves. It would apply to actual tubes only if the μ's of the individual grids were all infinite. Fortunately, a situation somewhat equivalent to this can be created at extremely high frequencies, but to describe it we must first consider relations between the various potentials at the equivalent grid planes and those on the grid wires themselves.

In doing this, the first thing to notice is that the potential difference between any grid plane and the corresponding grid wires may be expressed by an equation which has the general form of (5) on Table I and that the velocities of the electrons entering the differential region around the grid wires must be the same as of those which pass through the grid. Also, if the conduction current which passes through the grid is αq, then that moving toward the grid is $(1-\alpha)q$. For this region between grid plane and grid wires, the transit angle is extremely small, and it is accordingly appropriate to use (5) as a formula for calculation with the transit angle allowed to approach zero. The result for any grid is therefore an equation in the form of (21) for the corresponding region where however, $(1-\alpha)$ replaces α in the coefficients (22) and the transit angle approaches zero. Thus, for the control, or first, grid, the second equation of (21) gives the required form and we have

$$I_g = (V_g - V_1)y_g - V_1 y_{1g} \qquad (41)$$

where y_g is an admittance between the equivalent grid plane and the grid wires, and y_{1g} is a transadmittance between region (1) and the region between the equivalent plane and the grid wires. For the next grid, or screen, we have to follow the form of the third equation of (21) giving

$$I_s = (V_s - V_2)y_s - (V_2 - V_1)y_{2s} - V_1 y_{1s} \qquad (42)$$

where the y's have analogous meanings to those in (41). In substituting the coefficients of (22) to find the appropriate values of the admittances above, it is allowable to disregard space charge on the basis of the same arguments used in arriving at (23) for the intergrid regions.

The admittances y_g and y_s are thus of the form of the coefficient A^* for no space charge, and accordingly represent pure capacitances, C_g and C_s, respectively. Their magnitudes cannot be predicted from the simple geometry of planar regions because the region now considered is an equivalent one representing current flow from the equivalent plane of the grid to the grid wires, and is in actuality far from planar in configuration. The important thing is that, as part of the connecting network between the equivalent grid plane and the grid itself, there exists a simple capacitance. From a different argument it will turn out that the magnitude of the capacitance may be very simply expressed in terms of the amplification (or screening) factor of the grid in question.

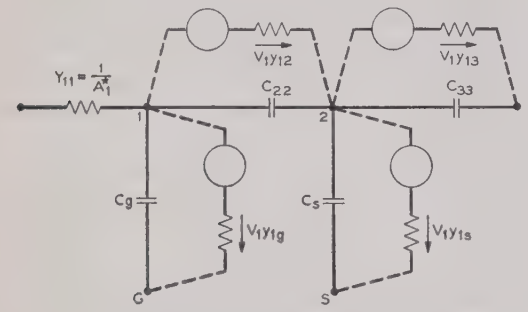

Fig. 11—General vacuum-tube network with space charge in region (1) only.

In regard to the transadmittances, y_{2s} is zero for the same reason that y_{23} is zero in (23) and the other transadmittances, y_{1g} and y_{1s} follow the forms of y_{12} and y_{13} respectively, in (23) where the only change necessary is to make B_2^* apply to the region between equivalent-control-grid plane and the control grid itself, and similarly to make B_3^* apply to the region between equivalent screen plane and the screen itself.

Substituting into (23) from Table II, and allowing the transit angle from the equivalent planes to the grids themselves to approach zero, we have

$$\left.\begin{aligned} y_g &= i\omega C_g, \qquad y_s = i\omega C_s \\ y_{1g} &= (1-\alpha_1)g_0\psi e^{-i\theta_1} \\ y_{1s} &= \alpha_1(1-\alpha_2)g_0\psi e^{-i(\theta_1+\theta_2)} \end{aligned}\right\}. \qquad (43)$$

In these equations $g_0\psi e^{-i\theta_1}$ is the function in the first column of the table following (25) and is plotted on Fig. 8.

The similarity of y_{1g} and y_{1s} to the forms of (31) and (32) is immediately apparent. The interpretation is that the electron stream causes a current to be impressed upon any region which it enters, and that when the stream passes the screen, for example, the conduction current splits into a fraction α_2 which proceeds into the screen-plate region and impresses on it a current which conforms to the characteristics of that region, while the remaining fraction $(1-\alpha_2)$ proceeds to the screen and impresses on the screen a current which conforms to the characteristics of the region between the equivalent plane of the screen and the screen wires. Because the transit angle through this latter region is extremely small its effect on the impressed current reduces to the factor unity (with a minus sign).

When the first grid is operated at a negative-bias potential, no electrons can hit it, and consequently the factor α_1 in (43), is unity. It results that y_{1g} is then zero, and the control grid is connected to the electron stream through a simple capacitance, C_g.

Even though the capacitances C_g and C_s have not yet been expressed in explicit form, we are in a position to draw the equivalent network of the entire vacuum tube in which the relation between the grids and the electron stream is taken into account in addition to the simpler network of Fig. 7, which expresses the electron stream only. This general vacuum-tube network is shown in Fig 11, where the admittances between the various grid planes are shown as simple capacitive elements in accord with the approximation that the space charge is negligible in those regions. The constant-current generators across the several elements impress currents upon the nodes at 1, 2, 3, G, and S in accord with the formulas developed above for the transadmittances. When the control grid is negative, the current impressed between 1 and G disappears, for then y_{1g} is zero since the capture fraction $(1-\alpha_1)$ in (43) is then zero. Again, in Fig. 11, only one current generator appears between nodes 2 and 3 instead of two generators, as in Fig. 7. This is because lack of space charge in the second region, between 1 and 2. has reduced the corresponding transadmittance y_{23} to zero.

The extension of the diagram of Fig. 11 to tubes with three, four, or more electrodes consists merely in inserting more sections similar in form to the one between nodes 1 and 2 with corresponding impressed currents and capacitances connecting the inserted region with the corresponding grid.

The current generators in shunt with the various regions in Fig. 6 could, if we wished, be replaced by voltage generators located in series with the capacitances C_{22}, C_{33}, C_g, and C_s. In this form the generators would have more the appearance of the μ generators of our conventional vacuum-tube equivalent circuits. They would be less convenient to use however, and would not be the same as our usual μ generators in any event. One of their inconvenient properties would be that the generated voltage would be required to approach infinity as the frequency approach zero. This is evident when it is considered that an infinite voltage is required to send a given current through a capacitance at zero frequency.

All in all, the admittance diagram of Fig. 11 with its internal nodes and impressed currents has been found to have many advantages over the conventional impedance diagram with meshes referred only to the available external terminals of the tube.

The capacitances C_{22}, C_{33}, etc., are merely the electrostatic capacitances which would exist in free space between solid conducting planes coinciding with the respective grids or plate as the case may be. The capacitances C_g, C_s between the electron stream and the grid

wires are related to C_{22} and C_{33} by the low-frequency screening factor of the corresponding grid, so that $C_g/C_{22}=\mu_g$ and $C_s/C_{33}=\mu_s$. Direct application of the analysis based on the equivalent network of Fig. 11 will now be applied to show how this comes about as well as to find the equivalent potentials of the internal nodes.

General Network

The generalized circuit network for a normal pentode where the space charge is small except near the cathode is illustrated in Fig. 12 where the control grid, screen, and suppressor are labeled, respectively G, S, and T. The plate is identical with node 4 and therefore does not need an additional symbol. Current directions are indicated by arrows in the usual way, and the method is to sum up the currents entering and leaving each of the three nodes 1, 2 and 3. Thus we have

$$\left.\begin{aligned} &\text{for node 1,} && I_g + I_b = I_a + V_1y_{12} + V_1y_{1g}\\ &\text{for node 2,} && I_s + I_c + V_1y_{12} = I_b + V_1y_{13} + V_1y_{1s}\\ &\text{for node 3,} && I_d + I_t + V_1y_{13} = I_c + V_1y_{14} + V_1y_{1t} \end{aligned}\right\} \quad (44)$$

and for the various currents

$$I_a = V_1y_{11}, \quad I_b = (V_2 - V_1)y_{22}, \quad I_c = (V_3 - V_2)y_{33},$$
$$I_d = (V_4 - V_3)y_{44}, \quad I_g = (V_g - V_1)y_g,$$
$$I_s = (V_s - V_2)y_s, \quad I_t = (V_t - V_3)y_t.$$

When these are substituted into (44) it is convenient to set up the resulting equations in the form of an array. Thus

V_1	V_2	V_3	
$(y_{11} + y_{22} + y_g + y_{12} + y_{1g})$	$-y_{22}$	0	V_gy_g
$(y_{13} + y_{1s} - y_{12} - y_{22})$	$(y_{22} + y_{33} + y_s)$	$-y_{33}$	V_sy_s
$(y_{14} + y_{1t} - y_{13})$	$-y_{33}$	$(y_{44} + y_{33} + y_t)$	$(V_ty_t + V_4y_{44})$.

(45)

Before proceeding with a discussion of the general solution of these equations, we can gain a better insight

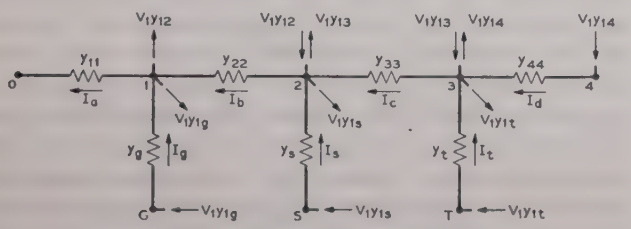

Fig. 12—Generalized pentode network with space charge in region (1) only.

into their significance by taking the first one alone. The equation is

$$V_1(y_{11} + y_{22} + y_g + y_{12} + y_{1g}) = V_gy_g + V_2y_{22}. \quad (46)$$

It applies directly to a triode as shown in Fig. 13, where V_2 is the plate potential. Dividing through by y_{22} and solving for V_1 we have

$$V_1 = \frac{V_2 + (y_g/y_{22})V_g}{1 + y_g/y_{22} + (y_{11}/y_{22})[1 + (y_{12} + y_{1g})/y_{11}]} \quad (47)$$

which bears a certain resemblance to the approximate formula which was mentioned earlier for the effective potential of the grid plane, namely,

$$V_1 = \frac{V_2 + \mu V_g}{1 + \mu + (4/3)(x_2/x_1)}. \quad (48)$$

To show that (48) is, in fact, an approximation for the more accurate form (47) we have to write the admittances in terms of the coefficients given by (23) and then substitute the forms given on Table II. It is at once evident that the ratio y_g/y_{22} may be taken as the very general form for the amplification factor μ, and hence that

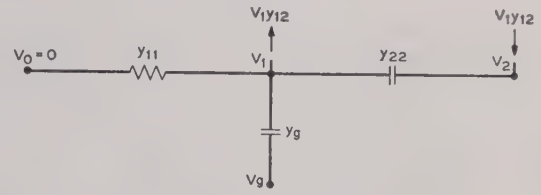

Fig. 13—Triode equivalent network.

in the special case

$$\mu = y_g/y_{22} = C_g/C_{22} \quad (49)$$

which allows us to find C_g in terms of μ and vice versa.

Then, at very low frequencies and for negative-grid tubes, the form of (47) becomes

$$V_1 = \frac{V_2 + \mu V_g}{1 + \mu + \frac{4}{3}\frac{x_2}{x_1}\left[1 + \frac{1}{2}\frac{\theta_2}{\theta_1}\frac{\sqrt{V_{D1}} + 2\sqrt{V_{D2}}}{\sqrt{V_{D1}} + \sqrt{V_{D2}}}\right]} \quad (50)$$

which shows the extent of the correction which should be made to (48). The approximation involved in (50) is the disregard of space charge between grid and plate. Its inclusion results in an expression which has been given before[5] but which may be written more conveniently in terms of the space-charge factor ζ_2 of the second region as follows:

$$V_1 = \frac{V_2 + \mu' V_g}{1 + \mu' + \frac{4}{3}\frac{x_2}{x_1}\frac{1}{1 - \zeta_2/3}\left[1 + \frac{1}{2}\frac{\theta_2}{\theta_1}\frac{\sqrt{V_{D1}} + 2\sqrt{V_{D2}}}{\sqrt{V_{D1}} + \sqrt{V_{D2}}} - \zeta_2\left(1 - \frac{\theta_2}{2\theta_1} + \frac{\theta_1}{2\theta_2}\right)\right]}. \quad (50a)$$

Here μ' is still given by y_g/y_{22} but is no longer equal to C_g/C_{22} as in (49). Instead, its general form for low frequencies and with negative-control grid is

$$\mu' = \frac{y_g}{y_{22}} = \frac{C_g}{C_{22}}\left(\frac{1-\zeta_2}{1-\zeta_2/3}\right). \tag{49a}$$

Also, in evaluating the transit-angle ratio θ_2/θ_1 in (50a) care must be taken to include space charge in the second region, since it modifies θ_2.

It is interesting to note that the modification produced by space charge in the second region is of the same form as that affecting the last term of the diode impedance (16).

At high frequencies, y_{22} in (49a) becomes modified and, for rather large values of ζ_2 the amplification factor then takes on the properties of a complex number having a phase angle. At extremely high frequencies, it again becomes real and is given by (49). While interesting from an academic point of view, these properties seldom have very much practical importance in conventional tubes where (49) applies quite well.

It is important to notice that neither (50) nor (50a) involves any factors which are different at zero frequency from their values at extremely low frequencies, and they contain no terms such as g_0 involving slopes of static characteristics. It follows that (50) gives the effective potential of the grid plane for direct current as well as for low-frequency alternating current. Accordingly, it is extremely useful in design work where the current to be drawn from a given cathode is limited by the emission capabilities of the thermionic surface, and hence, for a given location of the grid, the applied potentials V_{D0} and V_{D2} must be selected so that the effective potential V_{D1} will be low enough so as not to draw the entire emitted current away from the cathode and hence destroy the space-charge control.

It is true that at direct current the potential V_1 in (50) becomes V_{D1} and the equation then involves V_{D1} in a complicated manner, but the term involving it on the right-hand side of the equation is not large, and it is sufficiently accurate to use (48) to find a first approximation to V_{D1} and then to use this approximate value in the right-hand side of (50) to obtain a more accurate calculation. Similar considerations apply to (50a).

When the frequency is increased enough so that the transit angles become important, a further correction is needed in (50). However, all of the transit-angle terms appear in the correction term of (50) and hence the equation is a fair approximation for all moderately low frequencies as it stands. At extremely high frequencies, substitution from Table II into (47) together with the appropriate approximations shows that the effective potential then approaches:

$$V_1 = \frac{V_2 + \mu V_g}{1 + \mu + x_2/x_1}. \tag{51}$$

This is just the value that would be found for the effective potential in case there were no electrons whatever within the tube. Comparison with (50) shows the small range of values through which the effective potential changes, both as the frequency changes from zero to extremely large values and as the space charge changes from completeness at the cathode to zero in all regions.

Based on this fact, the proposal has sometimes been made that an approximate equation of the form of (48) be used in the general analysis of the performance of vacuum tubes.[14,15] However, in the light of the discussion which brought us to the equivalent network of Figs. 11 or 12 it seems better to use the diagram directly, regarding it as part of the complete circuit where the various external-circuit-impedance elements are connected to the tube terminals, and general circuit analysis is employed to give the currents and voltages in all of its parts as desired. Moreover, the simple relation which gives the effective potential in the form (48) is useful only when the potential V_2 is known. In triodes, this is the case, for V_2 then denotes the anode potential. For screen tubes, it is the effective potential of the screen, and the more comprehensive set of equations given by the array (45) must be resorted to when the various equivalent potentials are desired with a degree of exactness.

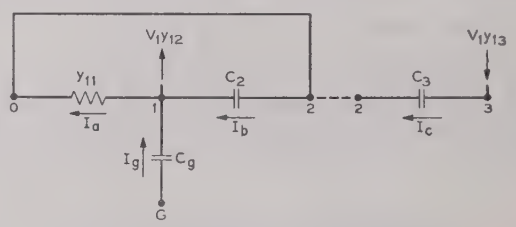

Fig. 14—Approximate network for screen tube with negative-control grid.

A complete solution of (45) is quite complicated when the coefficients of Table II are substituted to give the explicit forms of the y's. A great simplification occurs without much loss of accuracy when the admittances are evaluated in their very high-frequency forms. When this is done, the transadmittances are all found to be negligibly small compared with the self-admittances. To show the extent of the approximation involved, it may be remarked that it would yield (51) instead of (50) when applied to the triode. The arbitrary introduction of $(4/3)x_2/x_1$ for x_2/x_1 in the result would make the approximation even better. The ratio of the other distances, such as x_3/x_2 for example, should not be changed for the factor 4/3 appears because of space charge near the cathode, and hence is connected with x_1 but not with any of the other x's where the space-charge is small.

Attempts to arrange the result to conform with the form (51) have not proved particularly helpful for the general multielement tube, although for tetrodes some gain may be obtained in interpretive ease. The general forms for the internal nodes V_1, V_2, and V_3 for pentodes at low frequency thus become, from (45),

[14] D. O. North, "Analysis of the effects of space charge on grid impedance," PROC. I.R.E., vol. 24, pp. 108–136; January, 1936.

[15] H. Zuhrt, "Die Leistungsverstärkung bei ultrahohen Frequenzen und die Grenze der Rückkopplungsschwingungen," *Hochfrequenz. und Elektroakustik*, vol. 47, pp. 79–88; March, 1936; and vol. 49, pp. 73–87; March, 1937.

$$\left.\begin{aligned}
V_1 &= \frac{V_4+\mu_3V_T+\mu_2\left(1+\mu_3+\frac{x_4}{x_3}\right)V_s+\left[\left(1+\mu_2+\frac{x_3}{x_2}\right)\left(1+\mu_3+\frac{x_4}{x_3}\right)-\frac{x_4}{x_3}\right]\mu_1V_g}{\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)\left(1+\mu_2+\frac{x_3}{x_2}\right)\left(1+\mu_3+\frac{x_4}{x_3}\right)-\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)\frac{x_4}{x_3}-\left(1+\mu_3+\frac{x_4}{x_3}\right)\frac{x_3}{x_2}} \\
V_2 &= \frac{\left[V_4+\mu_3V_T+\mu_2\left(1+\mu_3+\frac{x_4}{x_3}\right)V_s\right]\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)+\frac{x_3}{x_2}\left(1+\mu_3+\frac{x_4}{x_3}\right)\mu_1V_g}{\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)\left(1+\mu_2+\frac{x_3}{x_2}\right)\left(1+\mu_3+\frac{x_4}{x_3}\right)-\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)\frac{x_4}{x_3}-\left(1+\mu_3+\frac{x_4}{x_3}\right)\frac{x_3}{x_2}} \\
V_3 &= \frac{(V_4+\mu_3V_T)\left[\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)\left(1+\mu_2+\frac{x_3}{x_2}\right)-\frac{x_3}{x_2}\right]+\frac{x_4}{x_3}\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)\mu_2V_s+\frac{x_3}{x_2}\frac{x_4}{x_3}\mu_1V_g}{\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)\left(1+\mu_2+\frac{x_3}{x_2}\right)\left(1+\mu_3+\frac{x_4}{x_3}\right)-\left(1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\right)\frac{x_4}{x_3}-\left(1+\mu_3+\frac{x_4}{x_3}\right)\frac{x_3}{x_2}}
\end{aligned}\right\}. \quad (52)$$

These may be reduced to the forms applying to tetrodes by allowing μ_3 in (52) to become very large and discarding all terms of which it is not a factor. The potential V_T then becomes the plate potential V_3 of the tetrode and we have for the tetrode

$$\left.\begin{aligned}
V_1 &= \frac{V_3+\mu_2V_s+(1+\mu_2+x_3/x_2)\mu_1V_g}{(1+\mu_1+4x_2/3x_1)(1+\mu_2+x_3/x_2)-x_3/x_2} \\
V_2 &= \frac{(V_3+\mu_2V_s)(1+\mu_1+4/3(x_2/x_1))+(x_3/x_2)\mu_1V_g}{(1+\mu_1+4x_2/3x_1)(1+\mu_2+x_3/x_2)-x_3/x_2}
\end{aligned}\right\}. \quad (53)$$

Allowable Approximations

These general forms are naturally fairly complicated. The same equations would be obtained on the basis of any kind of equivalent network, and the complexity is not a result of the particular equivalent network chosen, but of the attempts to take everything into consideration in the analysis of the tube performance. In dealing with pentodes and tetrodes in the past, we have seldom attempted to do this but have been content with certain approximations, such for example as assuming that the effective screen potential is near enough to that of the screen-grid wires so that they may be taken to be the same in calculating the effective potential of the control grid. The result, of course, is to change (52) or (53) into the simpler form (48) where V_2 is assumed to be the same as the known screen potential.

In analyzing the general performance of the tube when connected into an external circuit these statements have even greater force. Analysis of the entire equivalent circuit of Fig. 12 together with external-circuit connections would be an extremely tedious affair, and the result would include many effects which are discarded in ordinary low-frequency analysis as being negligibly small. For instance, in finding the performance of pentodes and tetrodes in amplifier circuits it is practically never necessary to include the effect of the capacitance between plate and control grid because it is so small that no appreciable effect on the result follows from disregarding it altogether. In Figs. 11 and 12 this capacitance feedback is naturally included in the complete diagram as shown even though it does not appear explicitly as a condenser connected between plate and control grid but rather as a coupling between the input and output circuits through the mutual admittances y_g, y_s, and y_t. When the screening factors are high enough, and when the screen and the suppressor are connected to the cathode, the diagram of Figs. 11 or 12 falls apart into two separate portions, the one involving only the input system and the other only the output system.

This is shown in Fig. 14 which illustrates the approximate equivalent diagram of a screen tube with negative-control grid. It will be found to be sufficiently accurate to handle all cases where the plate–control-grid capacitance may be neglected in the conventional diagram and where the screen is tied down to the cathode for alternating currents. The negative bias of the control grid has eliminated the impressed current V_1y_{1g} of Fig. 9 from node 1 in Fig. 14, and with the second node grounded, it is unnecessary to include any impressed currents on it.

With an output circuit of admittance Y_l connected between plate and cathode (that is, between plate and node 2) so that it is in shunt with the capacitance C_3, the output voltage is evidently given by the potential drop produced by the impressed current V_1y_{13} flowing through the parallel combination of load admittance and y_{33}, the admittance of C_3. The result is

$$V_{out} = V_3 - V_2 = V_1y_{13}/(Y_l + y_{33}). \quad (54)$$

Taking y_{13} from its detailed equation (25), we see from (27) that at low frequencies and for $\alpha_1=1$ it reduces to $(-\alpha_2g_0)$ where $(1-\alpha_2)$ is the capture fraction of the screen, and (54) then becomes, in terms of impedances rather than admittances,

$$\begin{aligned} V_{out} &= -V_1\alpha_2g_0\left(\frac{Z_l(1/i\omega C_3)}{Z_l+1/i\omega C_3}\right) \\ &= -V_1\alpha_2g_0Z_0 \end{aligned} \quad (55)$$

where Z_0 is the impedance of the parallel combination of C_3 and the external load Z_l. In this form a close analogy can be recognized to the usual equation

$$V_{out} = V_gg_mZ_0 \quad (56)$$

where g_m is the control-grid–plate transconductance. The difference between (55) and (56) is that the former is expressed in terms of the total transconductance $(-g_0)$, the screen "escape" factor α_2, and the potential at the node 1, while the latter is expressed in terms of the external transconductance g_m and the potential on the grid wires.

The relation between these two potentials has already been worked out and, for the grounded screen approximation now employed, reduces to (50) at low frequencies. With the screen and node 2 at ground, the alternating potential V_2 is zero, and we have V_1 immediately in terms of V_g. By substitution into (55) and comparison with (56) we have then

$$g_m = -\alpha_2 g_0 \frac{\mu_1}{1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\left[1+\frac{1}{2}\frac{\theta_2}{\theta_1}\frac{\sqrt{V_{D1}}+2\sqrt{V_{D1}}}{\sqrt{V_{D1}}+\sqrt{V_{D2}}}\right]}. \tag{57}$$

At high frequencies the external transadmittance from grid to plate is naturally more complicated, and the more general form (25) should be used for y_{13} in (54) to replace the low-frequency value $(-\alpha_2 g_0)$. Also, the relation between V_1 and V_g is not quite so simple as the one given by (50) and the general form (47) should be used. As explained before, however, this latter makes less difference than might be expected. However, including both of these effects, we can write the external transadmittance y_m as follows, first for moderately low frequencies where transit-time effects are just beginning to be important, and second for extremely high frequencies, where the transit angles are large compared with unity.

1. For moderately low frequencies

$$y_m \doteq g_m e^{-i\phi} \tag{58}$$

where

$$\phi = \frac{\left[\frac{11}{30}(\mu_1+1)+\frac{7}{18}\frac{x_2}{x_1}\right]\theta_1+\left[1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\left(1+\frac{1}{8}\frac{\theta_2}{\theta_1}\frac{3\sqrt{V_{D1}}+5\sqrt{V_{D2}}}{\sqrt{V_{D1}}+\sqrt{V_{D2}}}\right)\right]\theta_2}{1+\mu_1+\frac{4}{3}\frac{x_2}{x_1}\left[1+\frac{1}{2}\frac{\theta_2}{\theta_1}\frac{\sqrt{V_{D1}}+2\sqrt{V_{D2}}}{\sqrt{V_{D1}}+\sqrt{V_{D2}}}\right]}$$
$$+\frac{1}{3}\left[\frac{\sqrt{V_{D2}}+2\sqrt{V_{D2}}}{\sqrt{V_{D2}}+\sqrt{V_{D3}}}\right]\theta_3.$$

2. For very high frequencies

$$y_m \doteq +\alpha_2 g_0\left(\frac{\mu_1}{1+\mu_1+x_2/x_1}\right)e^{-i(\theta_1+\theta_3)}\left[\frac{\sqrt{V_{D2}}-\sqrt{V_{D3}}\,e^{-i\theta_3}}{i(\theta_3/2)(\sqrt{V_{D2}}+\sqrt{V_{D3}})}\right]. \tag{59}$$

Comparison of these with (27) and (35), respectively, shows the modification produced in attempting to relate the transadmittance to the external grid terminal of the tube rather than to the internal node at 1 on Fig. 14. The formulas, especially that for the phase of the transadmittance at moderately low frequencies, are much more awkward to handle. While the corrections are small, they may nonetheless be of importance in broadband-feedback-amplifier design. While it does not overcome the over-all calculation difficulties, it does simplify the qualitative interpretation to divide up the problem by referring the transadmittance to the internal nodes and then to make a separate calculation of the potential on the nodes in relation to that on the external grid terminal. This viewpoint is especially helpful when, as very often happens, the potential on the grid terminal is not known anyway, but the voltage somewhere back in the network connected to the grid is given and the grid voltage must be found by calculation. As shown by Fig. 14 in such a case it is just as easy to calculate the voltage at 1 as it is to calculate the voltage at G because the two points are connected by the simple capacitance C_g, which can be thought of as forming part of the external circuit.

Input Impedance

It becomes essential then in either case to know the input impedance of the tube. This will be calculated between the nodes 0 and 1, but may be referred to the grid immediately merely by adding the capacitive reactance $1/i\omega C_g$.

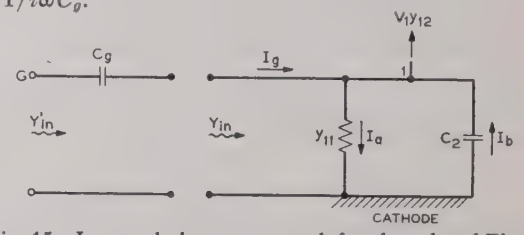

Fig. 15—Input admittance network for the tube of Fig. 14.

For calculating the input impedance between 0 and 1 the diagram of Fig. 15 is helpful. This is a redrawing of the pertinent portion of Fig. 14 with the capacitance C_2 shown in a clearer way for the impedance calculation. As before, complete space charge is assumed near the cathode and negligible space charge elsewhere. The effective screen potential is taken to be the same as that of the screen wires. At the node 1 we have the currents entering and leaving as follows: $I_g+I_b=I_a+V_1y_{12}$. In terms of voltages, and with the input admittance between 0 and 1 denoted by Y_{in} we have, since $i\omega C_2=y_{22}$

$$V_1Y_{in}-V_1y_{22}=V_1y_{11}+V_1y_{12}$$

or

$$Y_{in}=y_{11}+y_{22}+y_{12}. \tag{60}$$

At moderately low frequencies and at very high frequencies this may be evaluated with relative ease, as was illustrated in several foregoing cases, and we have

1. For moderately low frequencies

$$Y_{in} \doteq g_0 \frac{\theta_1^2}{20}\left[1 + \frac{22}{9}\frac{\theta_2}{\theta_1}\frac{\sqrt{V_{D1}} + 2\sqrt{V_{D2}}}{\sqrt{V_{D1}} + \sqrt{V_{D2}}} + \frac{5}{3}\left(\frac{\theta_2}{\theta_1}\right)^2 \frac{\sqrt{V_{D1}} + 3\sqrt{V_{D2}}}{\sqrt{V_{D1}} + \sqrt{V_{D2}}}\right]$$
$$+ i\omega\left[\frac{4}{3}C_1\left(1 + \frac{1}{2}\frac{\theta_2}{\theta_1}\frac{\sqrt{V_{D1}} + 2\sqrt{V_{D2}}}{\sqrt{V_{D1}}\quad\sqrt{V_{D2}}}\right) + C_2\right]; \tag{61}$$

2. For very high frequencies

$$Y_{in} \doteq i\omega(C_1 + C_2) - \frac{2g_0}{\theta_2}\left[\frac{\sqrt{V_{D1}}\sin\theta_1 - \sqrt{V_{D2}}\sin(\theta_1 + \theta_2)}{\sqrt{V_{D1}} + \sqrt{V_{D2}}} + \frac{3}{2}\frac{\theta_2}{\theta_1}\sin\theta_1\right]. \tag{62}$$

These represent a resistive element in shunt with a capacitive element between the node 1 and the cathode in Fig. 14. The capacitance C_g must be added in series to give the final grid-cathode impedance. Inasmuch, however, as C_g is normally much greater than C_1 and C_2 in parallel, the effect will be small, and (61) and (62) give a fair approximation to the input admittance as they stand.

As an alternative to adding the capacitance C_g to the equivalent network represented by (61) and (62) another method may be followed, which amounts to the same thing. This consists in noting that (47) expresses the general relation between V_1 and V_g and that V_2 is zero for the example of Fig. 14. Hence, since we can derive immediately from Fig. 15 the relation $Y'_{in} = Y_{in}(V_1/V_g)$ it follows that we have only to multiply the admittance between cathode and node 1 by the ratio V_1/V_g in order to find the external admittance between cathode and grid. As shown before, that ratio is given with fair accuracy by (50) for low frequencies and by (51) for very high ones, although to be exact, one more term in the series expansions leading to (50) should have been retained as well as the next lower order reactive term in (51). The approximation therefore amounts to omitting a very small phase shift that occurs in translating the admittance from the node 1 to the grid wires.

Without the details of this transformation we can see directly from (61) and (62) what the character of the input admittance is going to be.

In the low frequency case, from (61) it is evident that a resistive component appears across the input which is proportional to a first approximation to the total transconductance g_0 and to the square of the frequency. This input loading has been discussed at some length in previous papers[1,5,14,15] and is one of the important causes of loss of amplification when the attempt is made to use tubes with cathode and screen tied together for frequencies where the transit angles begin to be appreciable. Here again the remarks following equation (15) apply with special force, and the effect of the virtual cathode is to add a resistive component to the input impedance, which becomes especially important at the higher frequencies. It should be remarked, also, that even a small amount of inductance in the cathode lead between the point where the screen is tied in and the actual thermionic surface will act to increase the input conductance to values markedly greater than are predicted from (61) alone.

This whole question of impedance in the leads to the various tube elements is extremely important[16] at high frequencies, and a large part of the present-day advances in the construction of vacuum tubes for high frequencies and for broadband amplification has been obtained by reducing the lead lengths and arranging them to decrease both electrostatic and electromagnetic couplings.

In regard to the input conductance at very high frequencies, (62) shows that it can become either positive or negative, or zero depending upon the relative biasing voltages and transit angles. As a rough approximation, it may be assumed that θ_2 is a good deal less than θ_1 and that V_{D2} is much greater than V_{D1}. The high-frequency conductance then attains a value which is roughly given by $(2g_0 \sin\theta_1)/\theta_2$, which decreases as the frequency increases. At the same time the susceptance increases, which would indicate that the input losses may be very low at the higher frequencies, or may even become negative. This condition is somewhat modified, however, by losses occasioned between the cathode and the potential minimum, so that practically the input losses will be greater than predicted by the simple theory.

Amplification by Velocity Variation

The methods of analysis developed in the preceding pages are applicable to a method of high-frequency amplification[17,18] which has been called "velocity modulation" though "velocity variation" would more aptly describe the principle, and will be used hereafter in this paper. In that method, electrons are accelerated from a cathode and are shot at high velocity through an input cavity and then through an output cavity, with the space between the two great enough so that the transit angle between exit from the input cavity and entrance to the output cavity is fairly large. The velocity-variation method differs from that described for space-charge control in that the electrons enter the input cavity at high velocity while in space-charge control they start from the cathode with very low velocity.

In adapting the mathematical analysis which has been used here for the space-charge control to the requirements of velocity variation, the fundamental forms of Tables I and II require no change, except perhaps the comments that in long drift tubes of the type sometimes employed with velocity circuits, the parallel-plane assumption is not strictly applicable when the space-

[16] M. J. O. Strutt and A. van der Ziel, "The causes for the increase of the admittances of modern high-frequency amplifier tubes on short waves," Proc. I.R.E., vol. 26, pp. 1011–1033; August, 1938.

[17] W. C. Hahn and G. F. Metcalf, "Velocity-modulated tubes," Proc. I.R.E., vol. 27, pp. 106–117; February, 1939.

[18] D. L. Webster, "Cathode-ray bunching," *Jour. Appl. Phys.* vol. 10, pp. 501–508; July, 1939.

charge factor ζ is large. For small values of this factor, however, the relations are correct as they stand.

The schematic diagram of Fig. 5 is also pertinent as well as the network of Fig. 7. The general equations (21) and (22) apply but here the detailed similarity ceases and different initial conditions must be used for the evaluation of the A^*, B^*, C^*, etc., functions in which the admittances are expressed. Thus, instead of B_1^* in (5) being zero because the electron velocity at node zero is small we have the alternating conduction current equal to zero so that the term $B_1^* q_0$ in (5) drops out just as it did before, though for a different reason. Similarly $C_1^* v_0$ is zero because the initial alternating-current velocity is zero.

In applying the formulas of Table II to the evaluation of the admittances in (22) an enormous simplification can be made without loss of the fundamental principles of velocity variation if we assume that the space-charge factor ζ is very small in all regions involved, including the input cavity. With this assumption, all of the self-admittances, y_{11}, y_{22}, y_{33}, etc., in (22) become purely capacitive. The transadmittances y_{12}, y_{13}, y_{23}, etc., involve ζ at least once in each term, and hence ζ cannot be placed exactly equal to zero without loss of the energizing force in the velocity-variation system. However, it is evident that terms containing ζ^2, ζ^3, etc., may safely be disregarded in comparison with those which contain ζ to the first power only. Since C^*, D^*, and F^* all contain ζ as a factor, and moreover, are the only coefficients on Table II that do contain ζ as a simple factor, it follows that we can discard all terms which contain the products of any two of these coefficients, C^*, D^*, and F^*.

The only one of the transadmittances which is of interest to us in analyzing velocity variation is y_{13}. This is because we are not concerned with conditions between nodes 1 and 2 which alone involve y_{12} and because the potential between nodes 1 and 2 is practically zero (or the impedance infinite) so that $y_{23}(V_2 - V_1)$ does not introduce any effect into the output cavity.

With these stipulations, the transadmittance of a velocity variation tube is expressed by

$$y_{13} = \frac{\alpha_1 D_1^*}{A_1^*}\,\alpha_2 E_2^*\,\frac{B_3^*}{A_3^*} + \frac{G_1^*}{A_1^*}\,\alpha_2 F_2^*\,\frac{B_3^*}{A_3^*} + \frac{G_1^*}{A_1^*}\,I_2^*\,\frac{C_3^*}{A_3^*} \qquad (63)$$

where ζ may be set equal to zero everywhere except in the evaluation of C^*, D^*, and F^* where it must be retained. Upon substitution from Table II and after rearrangement in the same manner employed for space-charge control tubes, (63) may be written

$$y_{13} = -i\alpha_1\alpha_2\,\frac{I_D}{2V_{D2}}\left[\frac{V_{D2}}{V_{D1}}\left(\frac{\sqrt{V_{D1}}}{\sqrt{V_{D0}}+\sqrt{V_{D1}}}\right)\theta_1 + \sqrt{\frac{V_{D2}}{V_{D1}}}\,\theta_2 + \frac{\sqrt{V_{D1}}}{\sqrt{V_{D1}}+\sqrt{V_{D2}}}\,\theta_3\right] e^{-i\theta_2} \qquad (64)$$

where, according to usual practice, the transit angles θ_1 and θ_3 are taken to be small. When they are extremely small, and when, moreover, $V_{D1} = V_{D2}$ so that the electrons move through the space with constant velocity, (64) reduces to

$$y_{13} \doteq -i\alpha_1\alpha_2(I_D/V_{D2})\tfrac{1}{2}\theta_2 e^{-i\theta_2} \qquad (65)$$

which is the same equation that has been given by others[17] for the velocity variation system.

In all of the foregoing discussion, the question of noise has been left out of consideration. Its influence will, of course, have a most important bearing on the type of structure and the transit angles chosen for particular purposes. That is another question, however, and to deal with it adequately would require another paper perhaps longer than this one.

Synopsis of Principal Results

The basic geometrical configuration considered is shown in Fig. 1. The two parallel planes between which the electrons travel are marked *a* and *b* and the electron flow is taken to occur from left to right. It is emphasized that analysis throughout the paper is confined to single-valued velocity electron streams. As given conditions, are assumed the total current, the conduction current, and the electron velocity at the *a* plane in the figure. After separation of these quantities into direct- and alternating-current components, two sets of equations are given: (2) for direct current and (5) for alternating current. A new space-charge parameter ζ is defined and Tables I and II summarize the most important material pertaining to (2) and (5). Tables I and II are of fundamental importance for the paper.

The basic equations are first applied to a diode and the cases of complete and partial space charge are considered in greater detail. For complete space charge Figs. 3 and 4 show real and imaginary parts of the impedance and the admittance as functions of the transit angle. This diode impedance is composed of a resistance in series with a negative reactance. The resistance component may assume negative as well as positive values.

In the case of partial space charge the diode impedance may be broken up into three elements in series. The first is a resistive element, the second a negative reactance element, and the third a pure capacitance.

Multielement Tubes

With the diode background the building-up of the equivalent network may be undertaken. The method is to imagine several cascade arrangements of the parallel-plane diode geometry of Fig. 1. As an illustration of this method the tetrode of Fig. 5 is chosen. Between the cathode and control grid exist conditions similar to those in Fig. 1 when proper boundary conditions at the *a* and *b* planes are selected. Between control grid and screen another parallel-plane diode may be envisioned but with appropriate boundary conditions and a different value of the space-charge factor from that existing in the first-mentioned region. A similar diode is located between the screen and the plate. These diodes are to be joined together by relating the boundary conditions at the grid plane in terms of an effective potential. The grid wires themselves are at a different potential from this effective potential and the grid current is the difference between the total current flowing out of the left-hand surface of the fictitious plane and that flowing

into its right-hand surface. This is illustrated by the difference between the currents I_1 and I_2 in Fig. 5. Similar considerations apply to the other grids. Attention is first directed to the main electron stream assumed to originate at plane 0 in Fig. 5. For region (1) the diode equations apply directly. For region (2) equations (5) on Table I provide the means for calculating the initial conduction current and velocity so that currents and velocity at the right-hand end of region (2) can be calculated. For regions (3), (4), etc., a similar procedure is followed. The result is given by (21) and (22). From (21) the equivalent circuits of Figs. 6 and 7 follow. The constant-current generators shown on these figures play a role similar to the familiar constant-voltage μ generators. The impressed currents are expressed in terms of the voltages of the equivalent grid planes rather than in terms of the grid-wire voltages. As soon as the relations between the grid voltages and the equivalent grid plane voltages have been found the admittance coefficient y given by (22) can be extended to include the grid terminals of the tube. Before this is undertaken a more detailed analysis of the admittance coefficients (22) is desirable. The example chosen is a tetrode operated with complete space charge in region (1) and with so small a space charge in succeeding regions that the space-charge factor for these regions is very small. This operating condition corresponds to the usual one. The admittance coefficients (22) become simplified and are now given by (23). Detailed expressions for the transadmittances for any frequency are given by (24) and (25). For moderately low frequencies, (24) and (25) may be expanded and yield (26) and (27). These equations show that as far as the magnitude of the transadmittances is concerned the frequency has only a small effect. The most important effect of transit time in this frequency range is to cause a rotation in the phase of the transadmittance. Another case of interest occurs when the transit angle in all regions except the last is large while in the last it is small. Equations (31) and (32) apply for this case. The total phase of the transadmittance of a tetrode is shown in Fig. 9.

When all regions have large transit angles (24) and (25) degenerate into (34) and (35). The magnitudes of the transadmittances now follow fluctuating curves. Fig. 10 shows one example. Voltage gains for several conditions are given by (38) and (40).

Extension to External Terminals

The analysis so far would apply to actual tubes if the μ's of the individual grids were infinitely large. This not being the case, the existing relations between the potentials of the equivalent grid planes and those of the grid wires must be considered. Equation (5) on Table I furnishes the basis. In applying (5) it is assumed that between any equivalent grid plane and the corresponding grid the transit angle as well as the space-charge factor are very small. The admittance coefficients between the equivalent grid plane and the grid then assume the values given by (43). From (43) the interpretation follows that the electron stream causes a current to be impressed upon any region which it enters, and that when, for example, the stream passes the screen the conduction current splits into a fraction α_2 which proceeds into the screen-plate region and impresses on it a current which conforms to the characteristics of that region, while the remaining fraction $(1-\alpha_2)$ proceeds to the screen and impresses on the screen a current conforming to the characteristics of the region between the equivalent screen plane and the screen wires. The general vacuum-tube network now becomes that shown in Fig. 11. The admittances between the various grid planes are those of pure capacitances by virtue of the assumption of negligible space charge in these regions. The constant-current generators impress currents upon the nodes at 1, 2, 3, G, and S. When the control grid is biased negatively, the impressed current generator between 1 and G disappears.

The capacitances C_{22}, C_{33}, etc., are the electrostatic capacitances existing in free space between solid conducting planes coinciding with the respective grids or plate. The capacitances C_g and C_s between the electron stream and the grid wires are related to the capacitances C_{22} and C_{33} by the low-frequency amplification factors of the grids.

General Network

The principal object of this section is to calculate the effective potentials as well as to express the amplification factors as ratios of two capacitances. The equivalent network of Fig. 11 serves as a basis and a pentode is chosen for illustration. The equivalent circuit of the pentode is shown in Fig. 12 where the control grid, screen and suppressor are labelled G, S, and T, respectively. Direct application of the circuit equations results in (45) for the unknown effective potentials V_1, V_2, and V_3. As an introduction to the general case a triode is first considered and (47) gives the effective grid potential at any frequency for this type of tube and (49) gives the amplification factor as a ratio between the capacitances C_g and C_{22}. For moderately low frequencies and with disregard of space charge in the grid-anode region, (47) goes over into (50). Inclusion of space charge gives (50a) for the effective potential and (49a) for the amplification factor. It is important to observe that neither (50) nor (50a) contains any factors which are different at zero frequency from their values at moderately low frequencies. It thus follows that (50) holds both for direct and alternating current. At very high frequencies (51) gives the effective potential. A comparison with (50) shows that the effective potential passes through only a very small range of values as the frequency changes from zero to extremely large values.

The complete solution of (45) is quite complicated. Simplification and not much loss of accuracy occurs when the admittances are given their high-frequency forms. When this is done equations (52) are obtained.

Allowable Approximations

The general forms of the effective potentials are fairly complicated and it is natural to search for allowable approximations. It would, for example, be a tedious

and involved affair to analyze the entire circuit of Fig. 12. The result would, however, include effects which usually are discarded in ordinary low-frequency analysis. For instance, in finding the performance of pentodes or tetrodes it is rarely necessary to include the capacitance between plate and control grid. In Figs. 11 and 12 this capacitance is included although it does not appear explicitly. Furthermore, when the amplification factors are very large and when the screen and suppressor are tied to the cathode the circuits of Figs. 11 and 12 fall apart into two separate portions, namely, one involving only the input system and another only the output system. Fig. 14 illustrates the approximate equivalent circuit of a tetrode with negatively biased control grid.

With the equivalent circuit of Fig. 14 as a basis the control grid-plate transconductance is given by (57) for very low frequencies and at moderately low frequencies by (58). At extremely high frequencies it is given by (59).

Input Impedance

The input impedance of the tetrode of Fig. 14 is considered in this section. It is calculated between the nodes 0 and 1 but it may easily be referred to the grid by adding the reactance of the capacitance C_g. The general expression for input admittance between nodes 0 and 1 is given by (60). At moderately low frequencies it is given by (61) and at very high frequencies by (62). Both of these expressions represent a resistive element in shunt with a capacitive element and the capacitance C_g must be added in series to give the grid-cathode impedance. For moderately low frequencies the resistive component across the input is very roughly proportional to the total transconductance and to the square of the frequency. At very high frequencies the resistive component across the input may become either positive, negative or zero. The clean-cut results of the analysis are modified to some extent by the Maxwellian distribution of velocities of emitted electrons.

Amplification by Velocity Variation

As a further illustration of the generality of the methods, an expression for the transadmittance of the conventional planar velocity variation amplifier tube is given. The result (69) is identical with the usual formulas when the assumed conditions of operation are the same.

Appendix

Tables of Values Used in Figures 2, 3, 4, 8, and 10,

Fig. 2—Graph of Space-Charge Factor ζ

ζ	I_D/I_m
0.030	0.066
0.060	0.129
0.090	0.191
0.150	0.305
0.210	0.409
0.300	0.547
0.450	0.732
0.600	0.864
0.720	0.936
0.750	0.949
0.780	0.961
0.810	0.971
0.840	0.980
0.870	0.987
0.960	0.999

Fig. 3—Diode Impedance with Complete Space Charge

θ (Radians)	Resistance r/r_0	Reactance x/r_0
0	1.000000	0.000000
0.4	0.989379	−0.119240
1.0	0.935093	−0.288320
1.4	0.875997	−0.388551
$\frac{\pi}{2}$ = 1.570796	0.845991	−0.427248
1.8	0.801872	−0.474784
$\frac{3\pi}{4}$ = 2.356195	0.680630	−0.566900
2.8	0.575244	−0.615070
π = 3.141593	0.492768	−0.636620
3.6	0.384844	−0.645341
4.0	0.296930	−0.635892
$\frac{3\pi}{2}$ = 4.71239	0.163340	−0.587754
5.2	0.092785	−0.538847
5.6	0.048612	−0.493874
2π = 6.283186	0.000000	−0.415065
6.8	−0.017391	−0.359815
7.2	−0.022019	−0.322399
$\frac{5\pi}{2}$ = 7.853980	−0.018462	−0.273109
8.4	−0.009979	−0.243708
9.0	−0.000206	−0.222267
3π = 9.424776	+0.006084	−0.212207
10	0.010942	−0.203237
$\frac{7\pi}{2}$ = 10.995572	0.010669	−0.192560
4π = 12.566372	0.000000	−0.171249

Fig. 4—Diode Admittance r_0/z with Complete Space Charge

θ (Radians)	Conductance	Susceptance
0	1.000000	0
0.4	0.996264	0.120070
1.0	0.976571	0.301109
1.4	0.953889	0.423100
1.5708	0.941830	0.475649
1.8	0.923370	0.546723
2.3562	0.867450	0.722505
2.8	0.811098	0.867252
3.1416	0.760321	0.982279
3.6	0.681659	1.143067
4.0	0.602872	1.291083
4.7124	0.438927	1.579412
5.2	0.310354	1.802374
5.6	0.197389	2.005379
6.2832	0.000000	2.409264
6.8	−0.134015	2.772729
6.8068	−0.135493	2.777068
7.0686	−0.189110	2.980716
7.2	−0.210857	3.087346
7.3304	−0.228153	3.195674
7.8540	−0.246388	3.644877
8.3776	−0.172792	4.079026
8.4	−0.167734	4.096403
8.6394	−0.106033	4.272694
8.9012	−0.027116	4.443148
9.0	−0.004170	4.499090
9.4248	+0.134984	4.708519
9.9484	+0.255859	4.891084
10.0	+0.264140	4.906143
10.2102	+0.290376	4.964417
10.4720	+0.306247	5.033428
10.9956	+0.286842	5.177297
11.5192	+0.214540	5.351020
11.7810	+0.165427	5.454252
12.0428	+0.111363	5.569853
12.5664	0.000000	5.839450

Fig. 8—Phase and Magnitude of the Function $(1/g_0)(2P_1/\beta_1^2A_1^*)$

θ (Radians)	Magnitude	Phase (Radians)
0	1.0000	0
0.2618	0.9995	− 0.1038
0.5236	0.9983	− 0.1916
0.7854	0.9962	− 0.2892
1.0472	0.9933	− 0.3864
1.3090	0.9895	− 0.4849
1.5708	0.9847	− 0.5843
1.8326	0.9792	− 0.6856
2.0944	0.9727	− 0.7885
2.3562	0.9653	− 0.8936
2.6180	0.9570	− 1.0011
2.8798	0.9477	− 1.1118
3.1416	0.9374	− 1.2607
3.4034	0.9262	− 1.3434
3.6652	0.9139	− 1.4657
3.9270	0.9008	− 1.5931
4.1888	0.8867	− 1.7267
4.4506	0.8718	− 1.8672
4.7124	0.8561	− 2.0152
4.9742	0.8400	− 2.1724
5.2360	0.8244	− 2.3400
5.4978	0.8074	− 2.5192
5.7596	0.7920	− 2.7115
6.0214	0.7782	− 2.9185
6.2832	0.7668	− 3.1416
6.5452	0.7590	− 3.3804
7.0686	0.7611	− 3.9095
7.8540	0.8204	− 4.7897
8.6394	0.9292	− 5.6849
9.4248	1.0218	− 6.5209
10.2102	1.0542	− 7.2822
10.9956	1.0325	− 7.9926
11.7810	0.9822	− 8.6931
12.5664	0.9293	− 9.4248
13.3518	0.8951	−10.2163
14.1371	0.9088	−11.0476
14.9226	0.9542	−11.8878
15.7080	1.0081	−12.6990
16.4934	1.0391	−13.4701
17.2788	1.0360	−14.2161
18.0642	1.0062	−14.9517
18.8496	0.9673	−15.7080
19.6350	0.9404	−16.4984
20.4204	0.9400	−17.3175
21.2058	0.9670	−18.1399
21.9911	1.0041	−18.9426
22.7765	1.0295	−19.7175
23.5619	1.0311	−20.4745
24.3473	1.0110	−21.2248
25.1327	0.9814	−21.9911
25.9181	0.9586	−22.7820

Fig. 10—Magnitude of Transadmittance as a Function of Output Transit Angle

θ (Radians)	$\left\lvert\frac{\sin\theta/2}{\theta/2}\right\rvert$
0	1.00000
0.5236	0.98862
1.0472	0.95493
1.5708	0.90032
2.0944	0.82700
2.6180	0.73792
3.1416	0.63662
3.6652	0.52708
4.1888	0.41349
4.7124	0.30011
5.2360	0.19099
5.7596	0.08987
6.2832	0.00000
6.8068	0.07605
7.3304	0.13642
7.8540	0.18006
8.3776	0.20674
8.9012	0.21703
9.4248	0.21221
9.9484	0.19419
10.4720	0.16540
10.9956	0.12862
11.5192	0.08681
12.0428	0.04298
12.5664	0.00000
13.0900	0.03954
13.6136	0.07346
14.1372	0.10004
14.6608	0.11814
15.1844	0.12723
15.7080	0.12732
16.2316	0.11902
16.7552	0.10337
17.2788	0.08185
17.8024	0.05617
18.3260	0.02825
18.8496	0.00000

It is intended to present to the readers of the PROCEEDINGS OF THE I.R.E. material of timely instructional value. Accordingly there follows a republication of the major portions of a chapter in the book: "Principles of Aeronautical Radio Engineering" by P. C. Sandretto, former Superintendent of the Communications Laboratory of the United Air Lines Transport Corporation, and now Lieutenant Colonel in the Air Corps, United States Army. Permission to republish this material has been courteously granted by the author and by the publisher, the McGraw-Hill Book Company, Inc., New York.

The Editor

Absolute Altimeters*

LT.-COL. PETER C. SANDRETTO†, SENIOR MEMBER, I.R.E.

Summary—**Recognizing the necessity for knowing accurately the distance between an airplane and the ground as a means for improving the safety of aerial navigation, workers in electronics and allied fields attempted for years to develop a successful device. A commercial model was finally developed in 1938. This article is a technical history of absolute-altimeter developments.**

SINCE the advent of aircraft, the method used for determining altitude has been the aneroid altimeter. This device consists merely of an expansible chamber geared to a pointer. This pointer deflects proportionally to the pressure exerted on the chamber by the surrounding air. Of course the deflection of this meter bears no direct relation to the distance between it and the terrain below. The meter is calibrated to read in feet, "standard" air conditions being assumed. An adjustment for the known conditions of the air is provided. At terminals the ground-station personnel advise the pilot by radio of the barometric pressure existing on the ground, and the pilot makes this adjustment on his altimeter. The altimeter then reads the approximate elevation of the airplane above sea level, and if the elevation above sea level of the ground directly below is known, it is possible by subtraction to determine the height of the airplane above the ground. In order to know accurately the distance between the airplane and the ground below, it is necessary to know, in addition to the reading of the meter, the air pressure and temperature. Consequently, it can be seen that the aneroid altimeter cannot be trusted as an indicator of the proximity of the adjacent terrain.

A reasonable question concerns the necessity for knowing this height with such great accuracy. Some of the first references to altimeters that would read the distance to the terrain below (known as absolute altimeters) mentioned their use as landing aids. When instruments were first used to make flights above and through overcasts, many accidents occurred because the airplane deviated from true course and collided with high-altitude terrain. Flying at that time was permitted at altitudes lower than the adjacent terrain, and often the course was through canyons where small deviations caused collision. Flying at altitudes sufficient to clear all adjacent terrain was not commonly practiced because of the characteristics of the older airplanes and their lack of oxygen equipment. Later, high-altitude flying became a rule, but the absolute altimeter was again considered as an independent means for checking instrument landing systems.

The necessity for an instrument of this character was recognized by inventors and many patents have been granted covering these devices. Currently, however, only one such apparatus unit is available commercially. Much of the work done on these devices followed three principles. These are the speed of sound and its reflection from a hard surface; the change in the specific capacitance of a condenser with the variation of the proximity of a conductor (the earth); and the speed of radio waves, together with their reflection by the earth. Commercial models that utilized two of these three principles were produced.

PRINCIPLE OF THE SONIC ALTIMETERS

The use of sound as a means for measuring distance followed from the successful use of this principle as a sounding device on boats (see Fig. 1). Briefly, it consists of a powerful sound generator from which sound is transmitted down from the airplane to the ground. This sound is reflected by the ground, and the returning echo acts upon a sensitive detector. The sound emitted from the generator is (with one exception) in the form of a short pulse so that it has ceased before the reflected sound reaches the detector. The third item in this instrument is a device that measures the interval between the time that the sound leaves the generator and reaches the detector. This indicator is calibrated to read directly in feet.

* Decimal classification: R526.4. Original manuscript received by the Institute, September 20, 1943. Published by permission from "Principles of Aeronautical Radio Engineering," by Peter C. Sandretto and the McGraw-Hill Book Company, New York, N. Y.

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Sound Generators

Because of the high level of the noise from the aircraft motors and propellers and the loss in intensity of the generated sound as it travels to the ground and returns, it is necessary that the initial sound intensity emitted by the generator be high. In order that the sound generator may be light and yet very powerful, special designs are required, and these designs have been the subject of much attention by the various inventors. Chemical, electrical, mechanical, and compressed gas supplies have been used for generating the sound.[1] In all but one design it was possible for the generator to store energy between pulses and release this energy in a single powerful burst.

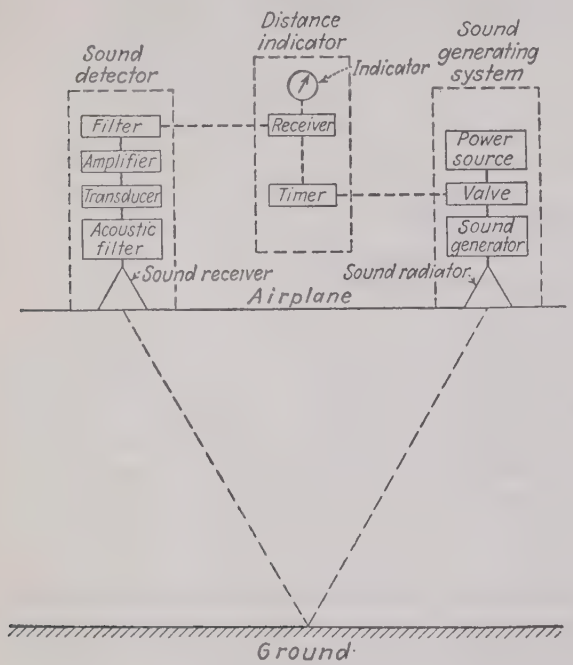


Fig. 1—Principal elements used in sonic altimeters.

In the "Behmlot" apparatus the transmitter consisted of a pistol that was fired at intervals. This system produced a very intense sound, yet the sound-generator weight was low, provided that only a small number of soundings was taken. This device was developed as early as 1924. Of course, the sound from this equipment had a random frequency range. The duration of this sound is not known, but it was emitted at 3-second intervals. In 1928, Nandillon worked on a sonic-altimeter development utilizing an armature-excited directive diaphragm without a horn. A constant frequency of 3500 cycles was emitted at intervals of about 0.005 second. These intervals were manually variable with the altitude. Rice of the General Electric Company produced the only near-commercial device in the United States. This unit was flight-tested by United Air Lines in 1933 and 1934, although work was done on it as early as 1929. Literature available states that compressed gas was bled from the engine cylinders and stored under pressure to be used later in actuating a whistle. However, the General Electric unit tested by United Air Lines used an electrically actuated hammer and anvil. The frequency of the whistle is reported as 3000 cycles per second for a duration of 0.01 second repeated at intervals of 2 seconds. This same characteristic applied approximately to the anvil generator. Work was also done in 1931 by Florisson of the Société de Condensation et d'Applications Méchaniques of Paris. His generator also consisted of a whistle and a conical horn. A small air compressor was carried for the purpose of supplying air for the whistle. The frequency of the whistle is not known, but it had a duration of about 0.03 second and was repeated at intervals of 1.1 seconds.

Dubois-Laboureur, in 1932, developed a device for the Constructions Électro-méchaniques d'Asnières using a siren with a constant frequency of about 1500 cycles per second. This sound had a duration of 0.013 second and was repeated at 0.7 second at low altitudes and 2 seconds at high altitudes. Also in 1932, Jacques-Badin worked with an electromagnetically excited diaphragm attached to an exponential horn. This system differed from the others in that the 200-cycle note was emitted continuously. In 1934, Delsasso used a mechanically excited diaphragm with a 2000-cycle note and a duration of 0.02 second. The "Echoscope" developed sometime prior to 1936 used a compressed-air-driven siren and parabolic horn emitting a 200-cycle note for a period of 0.02 second and repeated at an unknown interval.

Sound Detector

The devices that have been used for sound detectors on the various sonic altimeters are as many and as varied as the devices that have been used for sound generators. The Behmlot made use of a carbon microphone at the end of a horn, whereas Nandillon used an electromagnetic microphone. Literature reports Rice using a stethoscope, earpieces, acoustical filter, and horn, but the device tested by United Air Lines used an electromagnetic receiver, horn, wave filter, and amplifier. Essentially the same equipment was also used by Dubois-Laboureur and Jacques-Badin. The latter group secured further filtering by the use of an acoustical filter in addition to the electric filter, whereas the former used a tuned diaphragm on the microphone for the same purpose. Florisson used the system reported for Rice but added an additional acoustical filter. Delsasso used an electrical contact on a resonant diaphragm, whereas the Echoscope, like the Dubois-Laboureur device, used a tuned electromagnetic microphone. It can be seen that the means employed for detecting the reflected sound in the presence of the noise from the motor and propellers was the use of an audio frequency appreciably different from the major sound components of the airplane noise and filters for separating the generated frequency from the noise components. Because of the random frequency spectrum of the sound generated by the gun, a filter

[1] C. S. Draper, "The sonic altimeter," *National Advisory Committee for Aeronautics Tech. Notes*, vol. 611, August, 1937.

could not be applied to the Behmlot system. The fact that the Nandillon system is not described as using a filter is not significant because of the limited information on this secret development. All the other systems used filters. Three different types of filters—tuned diaphragm, electric, and acoustic—were employed by the Jacques-Badin device.

Distance Indicators

The distance indicators for the sonic altimeters are, of course, time-interval indicators calibrated in feet. Some of the devices used automatic indicators, and others required manipulation or continuous observation by the operator. In the early Behmlot system a dial rotating continuously at a constant speed was employed. The operator noted the time when he discharged the cartridge and again when he heard the reflected signal. Later this system was improved by connecting an optical attachment to the indicator. This optical system was quite ingenious. As the gun was fired, an electromagnet released a spring-acutated pointer which traveled across the scale at a constant speed. To this pointer there was attached a mirror which reflected light from a small lamp to a translucent scale. When the reflected signal was received, it was amplified and applied to an electromagnet which in turn actuated a mechanical reed. At the end of this reed was a lens positioned in front of the small lamp. The motion of the lens caused a deflection of the light beam and produced a positive indication on the translucent scale. Florisson used an indicator similar to that employed in the early Behmlot system, except that an auxiliary pointer was added to indicate the position of the timing hand when the sound was generated. This was later replaced with a light traveling across a scale, which was extinguished when sound was heard.

In the Nandillon system an indicator similar in principle to those described was used. The timing motor drove a lamp located at the end of an arm, at a constant speed. For one position of the lamp a contact was made which energized the sound generator. When the sound reached the detector, it was amplified and used to energize a magnet located below the lamp and actuating a shutter on it. The resulting spot of light was visible on a translucent scale in front of the lamp assembly. Indicators of this type are known as chronoscopes and were used in modified forms by Delsasso and in the Echoscope as well as by the investigators previously mentioned. In each case there is a constant-speed motor driving an indicator. When the returning sound wave reaches the detector, its presence is announced by a visual or aural sensation. The position of the indicator is noted, and its deflection from the starting point is proportional to time elapsed or distance to the ground. The indicator for the early Behmlot system was merely an aural indicator combined with a pointer, the Florisson used a light that was carried by the constant-speed motor and was extinguished, the Nandillon a light controlled by a shutter, the Delsasso a neon lamp illuminated by the signal, and the Echoscope a mechanical pointer actuated by an electromagnet which is deenergized by the returning sound.

The General Electric system tested by United Air Lines was intended primarily for use with instrument landing and did not have a direct indicator. The pilot noted the time that elapsed between the time when he heard the direct sound and the time when he heard the reflected sound, and it was intended that he learn to associate acoustically this time with altitude. The theory behind this procedure was that the pilot cannot know directly his altitude as he looks out but is, nevertheless, able to make a landing; hence he should be able to learn to associate his distance with an aural impression. By using a long tube between the receiving horn and the microphone of the detector, the accuracy of the instrument at low altitudes was increased, so when the pilot heard the direct and reflected signals coincide, he knew he had reached some predetermined terrain clearance.

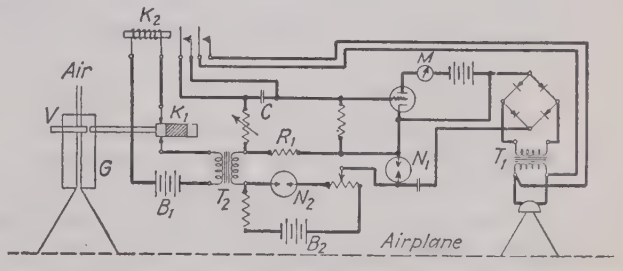


Fig. 2—Electronic indicator used with the Dubois-Laboureur sonic altimeter.

The Dubois-Laboureur system made use of an electronic indicator. The circuit for this device is shown in Fig. 2. In this circuit, G is the air-driven generator, which is a governor-controlled motor-driven siren. The motor that drives the siren also serves to operate the air valve V. The end of the valve shaft also serves as a commutator. Normally a battery B_1 is connected through the contacts K_1 of the valve shaft to the relay K_2. This causes K_2 to hold its contacts open, but when the valve shaft moves and furnishes air to the siren, the contacts at K_1 open and allow the relay contacts to close. These contacts short-circuit condenser C and microphone M. The momentary break in the current through the relay induces a voltage in the secondary of transformer T_2. This surge in the secondary is sufficient to cause the neon tube N_1 to conduct, allowing current to flow through R_1 from B_2. The second neon tube N_2 merely serves as a voltage regulator. The potential developed across R_1 causes the condenser C to be charged (after K_2 has again closed), and as the charge on this condenser grows, the plate current of the vacuum tube increases. When the sound is received by the microphone, it is rectified by the copper-oxide rectifier, and this voltage applied to N_2 causes the potential across it to be reduced, and N_1 stops conducting. This process is continuous, so the reading of the meter M is a function

of the average charge on the condenser C, which in turn is a function of the length of time between the period when C was discharged and when N_1 stopped conducting, that is, the period of time between the generation of the sound pulse and its detection.

A number of these indicators give only an intermittent indication. That is, the indicator gives a true reading only when the sound returns, after which the pointer, lamp, etc., are no longer energized. In the methods using light, an impression of continuous indication can be obtained if the pulses are sent out at a speed greater than the persistency of vision. Jacques-Badin used a system in which a single pulse is sent out, but the generator is not further energized until the signal returns. Upon returning, the received pulse automatically operates the switch controlling the energy to the sound generator, thereby sending out a second pulse. The rate at which these pulses are sent out is a function of the distance to the ground; that is, if the time between the sound transmission and reception is zero, the signal would be sent out continuously. The frequency at which these pulses are sent out is measured with a direct-reading frequency meter which is calibrated in feet.

Limitations of Sonic Altimeters

The most severe limitation of the sonic altimeter is due to the low speed of sound. As the speed of the airplane increases, the use of sound as a distance-measuring device becomes less practical and cannot be used when the airplane has reached the speed of sound. The comparatively slow speed of sound also limits the maximum useful altitude. On the assumption that there are no other problems, the length of time required to secure an indication is excessive at high altitudes. A modern transport airplane traveling at a speed that will be regarded as slow in the future covers a mile in about 20 seconds, yet this same amount of time is required for the sound to reach the ground and indicate in the cockpit of an airplane flying at the common altitude of 11,000 feet. This factor alone would render the sonic altimeter ineffective as an en-route flying device. Another factor limiting the usefulness of the sonic altimeter is the amount of power required in order to overcome the noise of the aircraft motors. If the sound transmitter is considered to be feeding its energy to a cone, the apex of which is the sound source, then the following equation may be written:

$$p = 134(\cos \delta / H)\sqrt{(W/1 - \cos \phi)} \qquad (1)$$

where H = height, feet
p = sound pressure, bars
W = sound power, watts
ϕ = half the angle of the cone of sound
δ = angle between cone axis and the vertical.

From this expression it can be seen that the angle of the cone of sound should be kept small; that is, the energy should be concentrated in as small an area as possible. For constant values of p, ϕ, and δ, the following equation may be written:

$$W \propto H^2. \qquad (2)$$

That is, the amount of power varies as the *square* of the height. Aside from these factors, sound is absorbed (for 300-cycle tones) at a rate of about $\frac{1}{2}$ decibel per hundred feet, and about 7 decibels are lost during the reflection at the ground. It can be seen that for high altitudes the sound power required alone renders the device impractical.

Performance of Sonic Altimeters

Sound powers of the order of 100 watts were used in some of the altimeters constructed. With this power, performance is reported for altitudes as high as 1400 feet. Without exception, readings at this elevation were made in airplanes with engines idling or in lighter-than-air craft with the motors shut off. The usual practical altitude with airplanes flying at high speed was more nearly 150 feet. The weight of these devices varied with the source of power employed. Weights reported varied from 20 to 61 pounds. The maximum altitude (for any condition) to gross-weight ratio varied from 8 to 48 feet of elevation per pound of weight.

Commercial Availability

The only unit advertised as commercially available in this country was the Rice-General Electric device. It is believed that this item was discontinued in about 1934. The Behmlot and the Echoscope were available in Germany in 1935 and the Florisson-SCAM and the Dubois-Laboureur-CEMA were available in France in 1933.

The Capacitance Altimeter

During the period when extensive development of the sonic altimeter was undertaken, work was also done on altimeters utilizing the capacitance principle. It appears, however, that most of this work was done by or associated with branches of the United States Services, and no extensive literature covering this subject has been published. It is not known whether Europeans worked along this line of endeavor.

The capacitance altimeter makes use of two conductors or plates mounted on a supporting structure outside the airplane. Electrically, then, these conductors will have capacitance to the structure of the airplane and to each other. This is shown in Fig. 3. The conductors

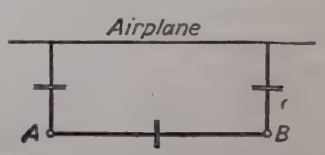


Fig. 3—Capacitances which exist between the electrodes of a capacitance-type altimeter when the airplane is in free space.

are A and B, and there exists a capacitance A to the airplane, B to the airplane, and between A and B. As the airplane arrives from free space to proximity with the ground, two other capacitances appear—one from

each conductor to ground as shown in Fig. 4. The three capacitances of Fig. 3 are equivalent to a single capacitance between the conductors, which equivalent capacitance is shown in Fig. 5 as C_{ABP}. The capacitances between individual conductors and ground can be grouped as C_{ABG}. The latter capacitance varies with the distance of the airplane from ground, and the development of the capacitance altimeter consists in devising an accurate means for measuring the change to this capacitance with change in altitude.

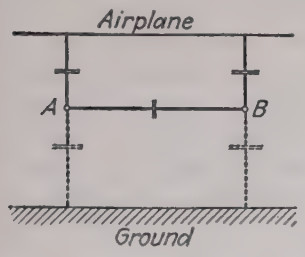


Fig. 4—Capacitances which exist between the electrodes of a capacitance-type altimeter when the airplane approaches the earth.

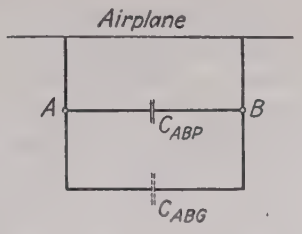


Fig. 5—Equivalent capacitances existing between the electrodes of a capacitance altimeter for the condition shown on Fig. 4.

Magnitude of Capacitance

If the diameter of the conductors forming the condenser plates in a capacitance altimeter is small compared with the length, the following formula[2] expresses the capacitance of these wires to ground:

$$C = \frac{0.2416L}{\log_{10}(2L/d) - k_2} \qquad (3)$$

where C = capacitance, micromicrofarads
L = length of conductor, centimeters
d = diameter of wire, centimeters
h = height above ground, centimeters
$k_2 = \log_{10}[L/4h + \sqrt{1+(L/4h)^2}]$.

Assuming that a wire having a diameter of 0.2 centimeter and a length of 250 centimeters is used for a conductor, the capacitance of this wire when 10 and 100 feet above ground will be 20.7 and 20.2 micromicrofarads, respectively. It can be seen that the capacitance change will be extremely small, and special means must be employed in order to detect these changes.

The Gunn Altimeter

An altimeter of this period was developed by Dr. Gunn of the United States Navy and is briefly described in the literature.[3] This device gave successful readings to altitudes of 100 feet, and with additional development it was thought possible to increase this altitude to 200 feet. A circuit[4] of the Gunn device is shown in Fig. 6.

Referring to this figure, a radio-frequency oscillator composed of coils L_1, L_2, tuning condenser C_1, and vacuum tube V_1 feeds power to an external circuit via coupling coil L_3. This energy is connected to two differentially wound coils L_4 and L_5. These coils are tuned by condensers C_2 and C_3 and couple energy to coil L_6. Since the voltage induced in coil L_6 is a function of the current in coils L_4 and L_5 and since these coils are identical and differentially wound, no voltage will be induced in L_6 if the currents in these coils are equal. The magnitude of this induced voltage is indicated by meter M of the vacuum-tube voltmeter composed of V_2, R_1, M, and the necessary batteries. Across coil L_5 there are attached two wires forming the external condensers. The condensers C_2 and C_3 are adjusted when the airplane is on the ground so that the voltage induced in L_6 is minimum. The effect of voltage remaining is canceled by an adjustment of the vacuum-tube voltmeter bias, which is controlled by potentiometer R_1. After the airplane leaves the ground, the capacitance across C_4 and C_5 decreases, thereby causing an unbalance in the currents because the current in L_5 decreases. A voltage is developed therefore across L_6 which causes a deflection in meter M proportional to the unbalance and, hence, proportional to the height. In order to accentuate this unbalance an additional tuned circuit consisting of coil L_7 and condenser C_6 is added in series with coupling coil L_3. This circuit is parallel resonant and, hence, keeps the current through L_4 and L_5 to a low value. Because of the high Q of coil L_7, changes in C_2 and C_3 produce detuning of this circuit and increase the current through L_4 and L_5, and, hence, the current unbalance.

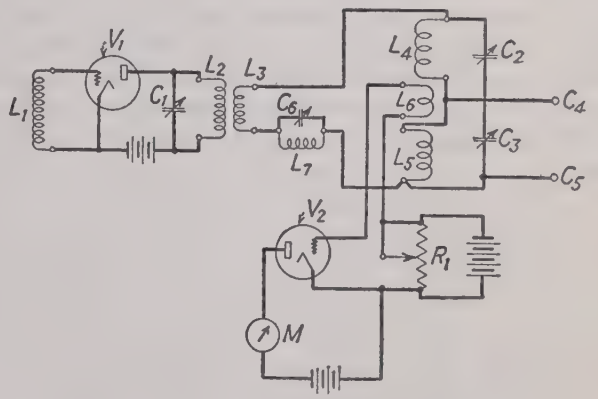


Fig. 6—The Gunn capacitance-altimeter circuit.

Results of Capacitance Altimeter Development

There seems to be no record of commercial-capacitance altimeter installations. Publications give the weight of one of these devices as 20 pounds. Obviously this weight is very low, so it could not have been the limitation of the device. Consideration of the problem involved indicates, however, that extreme stability, both electrically and mechanically, is necessary for proper operation of the device. Literature mentions the effect of wing flexure as a source of instability;

[2] "Radio instrument and measurements," *Bur. Stand. Circ.*, no. 74, second edition, p. 237; March, 1924.

[3] L. A. Hyland, "True altitude meters," *Aviation*, p. 1322; October 27, 1928.

[4] Ross Gunn, "Device for Indicating Small Changes of Capacity," U. S. Patent No. 1,701,975, February 12, 1929.

hence, all components must be very small in order that capacitance values remain fixed. It is doubted that these values could be maintained sufficiently stable for practical airline use where equipment must function day in and day out without attention by the designing engineers. Essentially, the device was satisfactory only as a landing aid, and was not suitable for en route flying. A device satisfactory for this purpose alone, but capable of withstanding service, would probably be somewhat heavier. Whether these considerations were the factors that prevented the capacitance altimeter from becoming a successful commercial product is not known, but apparently no commercial production was attempted.

Radio Altimeters

There are probably more patents covering various forms of radio altimeters than of any other absolute type. From all these patents, there is only one unit available on the market at this time, although the recent developments in microwave apparatus may introduce more. The unit now available or a modified form using microwaves may be adopted for commercial use in the future.

Early Radio Altimeters

Early radio altimeters attempted to measure altitude by sending out a radio signal which was reflected from the earth and by reading the intensity of the reflected wave. This system met with little success for two reasons. One was that the intensity was as much a function of the terrain over which the airplane was flying as it was of the height above the terrain. The second was that there were standing waves produced in space, so the signal sometimes decreased as the altitude decreased. The latter difficulty could have been corrected by using a very low frequency, but the effectiveness of the radiation on the airplane (for a maximum allowable size) decreases as the wavelength increases, and the first difficulty discussed would not have been corrected by this means. The solution worked out consisted in using still shorter wavelengths and counting the number of nodes and antinodes in the standing wave through which the airplane passed as it ascended or descended. To illustrate, if a wavelength of 100 feet were employed at the transmitter and the airplane began its ascent and passed through two nodes (regions of minimum signal) the distance traversed would have been equivalent to three quarters of a wavelength, or 75 feet.

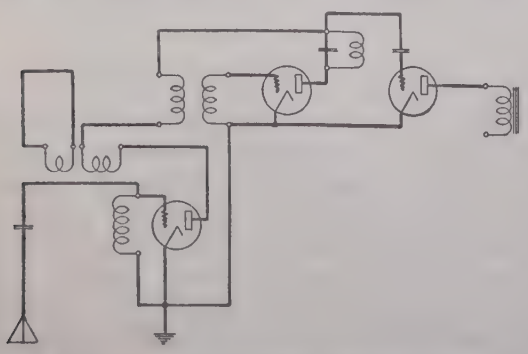


Fig. 7—Circuit of the early radio altimeter.

The circuit[5] of Fig. 7 shows the apparatus used with one form of this device. This apparatus consists largely of a regenerative receiver which serves as a transmitter as well as a receiver. The phase of the energy received serves to change (in a sinusoidal manner with altitude) the frequency of the device. This change is heard in the headphones. Increase and decrease of frequency as a function of altitude are shown in Fig. 8. Notice that

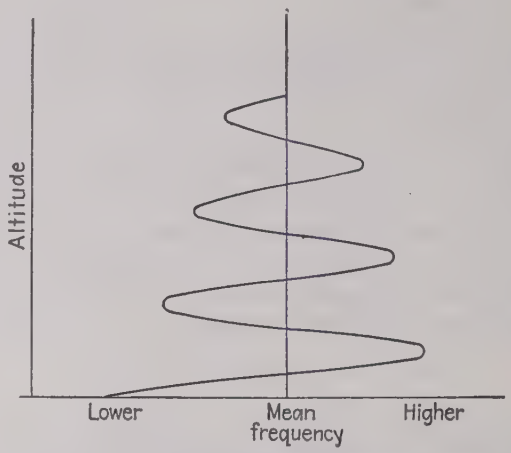


Fig. 8—Change of frequency with altitude which occurs for the radio altimeter of Fig. 7.

the amount of deviation of the frequency from the mean also varies with altitude. This is because the amount of energy that returns decreases with altitude and, hence, affects the amount of deviation. The transmitted and received energies are separated by using a loop antenna that may be oriented for minimum direct pickup. The disadvantage of this system is immediately apparent because it is necessary for the pilot to remember the number of nodes through which the airplane has passed in the process of ascending and descending.

The Alexanderson Altimeter

A device using a modified form of the principle described above was developed by Dr. Alexanderson of the General Electric Company and was widely reported in many periodicals in 1928 and 1929. In this device,[6] Alexanderson attempted to develop a mechanical "memory." Like many other altimeters of this period, its use as a means for landing under low-ceiling conditions was the major purpose of the development.

Two oscillators were used in this device, their output frequencies beating together in a detector. These oscillators are shown as O_1 and O_2 in Fig. 9. Oscillator O_1 is connected to an antenna A. As the reflected wave reaches this antenna the frequency of O_1 varies, as has

[5] "Radio altimeters," *Science and Invention*, vol. 16, pp. 952–953; February, 1929.

[6] Ernst F. W. Alexanderson, "Method and Means for Indicating Altitude from Aircraft," U. S. Patent No. 1,913,148, June 6, 1933.

previously been described. This frequency beats with a fixed frequency from oscillator O_2 in detector D. Actually O_2 has three frequencies that may be selected at will by manipulating control K. As K is moved, three windows of different colors are placed over light L. Transformer T is tuned to some frequency higher than the normal beat frequencies. This transformer is, in fact, a frequency discriminator and passes current proportional to frequency. This current is used to charge condenser C via rectifier R. This rectifier is inserted so that the charging mechanism will not discharge C even though, for the moment, there is no voltage from T. Thus, each time that a high-frequency peak (Fig. 8) is present, C charges and retains the charge. Each succeeding peak adds charge to C. As the charge on C increases, the plate current of the vacuum-tube voltmeter V increases and is recorded on the meter M. This plate current is normally zero because the grid of the tube is

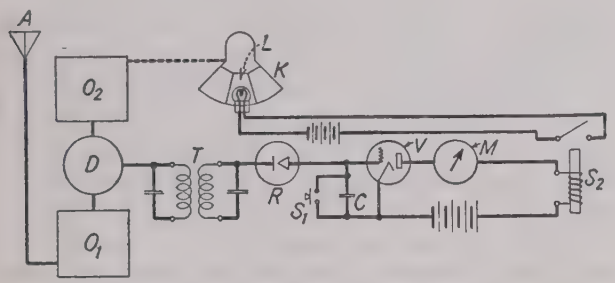


Fig. 9—The Alexanderson radio altimeter.

normally biased to the cutoff point by a bias battery (not shown). Thus, in a step-by-step process, meter M reads altitude. When the current flowing through M reaches a given value, the relay S_2 operates, thereby causing light L to be illuminated. This occurs at the point where M is reading full scale. At this time the pilot changes the frequency of O_2, thereby increasing the range of the instrument. He must also press switch S_1 to discharge condenser C.

Apparently no commercial exploitation of this device was attempted; at least its sales were not publicized. The reason it was not installed extensively on aircraft was not discussed in contemporary literature. The maximum altitude of the device was between 3000 and 4000 feet, and this would appear to have been sufficient to make the device worth while. The weight involved is not known, but it probably was not excessive. A study of the instrumentation involved leads to the conclusion that excessive attention was necessary on the part of the pilot. Whether or not this conclusion is correct is not known.

History of the Western Electric Radio Altimeter

This device, sometimes called the "terrain-clearance indicator," is herein designated by the term "Western Electric" because it is being currently manufactured and sold by this concern. The responsibility for its development rests on a large group of individuals and several organizations. Actually, all these should be credited for its ultimate production. The fact that a unit accomplishing the feat attempted by so many previous experimenters was finally produced certainly is attended with honor sufficient for all involved. No doubt other devices (possibly more successful) will soon reach the market, but there is no denying that this unit was the first to be successfully demonstrated and sold on a commercial scale.

The commercial development of this device can be traced back to work done by Professor Everitt of Ohio State University in 1928 and 1929 under a grant from the Daniel Guggenheim Fund for the promotion of Aeronautics.[7] In this work the principles later utilized in the Western Electric device were fully developed. No successful commercial model resulted from this work, however, largely because of the radio frequency used and because the grant obtained from the fund was exhausted. Professor Everitt realized at that time the limitation of the frequency he employed, but current vacuum-tube technique did not permit the generation of appreciable power at very high frequencies. In 1930, Lloyd Espenchied of the American Telephone and Telegraph Company applied for a patent[8] on a device somewhat similar to that used by Everitt. The original application was divided in 1936, and a patent covering this device was issued in the same year. The extent of the development work done under this patent prior to 1937 is believed to be only a mathematical analysis by the Bell Telephone Laboratories (associated with the American Telephone and Telegraph Company). In the meantime, R. C. Newhouse, one of the students who had worked on the altimeter under Everitt, was employed by the Bell Telephone Laboratories. The Communications Laboratories of United Air Lines were familiar with Newhouse's work and in 1937–1938 negotiated with the Western Electric Company for the development of such a device. Development work for the Western Electric Company is done by the Bell Telephone Laboratories, which organization had by this time developed tubes capable of producing appreciable power at frequencies in excess of 900 megacycles. Experienced personnel, equipment, and a patent were all available, so a successful model was developed and demonstrated to the public in the United Air Lines Laboratory airplane in the Fall[9] of 1938.

Principle of Weco Altimeter

The principle of this altimeter can best be described by considering the space between the airplane and the ground as a two-wire transmission line with its end open. If a voltage is connected across one end of the line, an electric wave will travel down it, reach the open point,

[7] "Solving the Problem of Fog Flying," Daniel Guggenheim Fund for the Promotion of Aeronautics, p. 29; 1929.

[8] Lloyd Espenschied, "Method and Means for Measuring Altitude of Aircraft," U. S. Patent No. 2,045,072, June 23, 1936.

[9] Henry W. Roberts, "The absolute altimeter," *Aero Digest*, p. 87; November, 1938.

and be reflected back to the source of the voltage. An appreciable length of time is required for this wave to travel from the voltage source to the open end of the line and back again, so when it reaches the source its phase will not be the same as the phase of the source voltage. This voltage will then add vectorially to the source voltage, making the terminal voltage either greater or less than the voltage of the generator when it is not connected to the line. This voltage, for a given length of line and for generators of the same characteristic impedance and open-circuit voltage, will vary as a function of the frequency employed. This fact follows because (although the time required for the electric wave to travel to the open terminal and return is the same for two different frequencies) if one of these is half the frequency of the other, the voltage at the lower frequency will rise from zero to some finite maximum peak value in the same time that the higher-frequency voltage rises to a maximum and again decreases to zero. If the generator is made with a variable-frequency control and adjusted first to a frequency that gives maximum voltage then to the next successive frequency again producing maximum voltage, it will be found that the difference between the two frequencies corresponds to an electrical length of one-half wavelength.[10] If the distance to the point of reflection is D, then

$$D = a\lambda_1 = aV_1/f_1 \tag{4}$$

where a = a constant
λ_1 = wavelength corresponding to f_1
f_1 = first frequency
V_1 = velocity of propagation along the transmission line

If the second frequency at which maximum voltage was observed is f_2, then

$$D = (a + 1/2)\lambda_2 = (a + 1/2)V_2/f_2. \tag{5}$$

In this equation a second velocity of propagation for the second frequency is written as V_2. Substituting (4) in (5),

$$D = (Df_1/V_1 + 1/2)V_2/f_2. \tag{6}$$

Solving for D in terms of both frequencies

$$V_1V_2/D = 2(V_1f_2 - V_2f_1) \cdot \tag{7}$$

For air V_1 and V_2 will be equal to each other and equal to c, the velocity of radio waves in space; therefore

$$D = c/2(f_2 - f_1) \cdot \tag{8}$$

This equation means, then, that if two arithmetically successive frequencies are known for which the voltages present at the terminals are the same, the length of the transmission line (or the space from airplane to ground) may be calculated.

Suppose that the difference between these "measuring" frequencies is allowed to remain the same for any value of line distance equal to nD, then,

$$nD = 0.5c/f_d/n \tag{9}$$

where f_d is the difference frequency equal to $f_2 - f_1$.

This expression means that only one nth of the previous frequency difference is required to measure a distance n times the length of that previously measured. Or, in other words, for the same frequency difference, there will be n times more voltage peaks at the generator terminals.

For a known frequency difference, then, it is possible to know the distance merely by sweeping the voltage generator between two known frequencies and counting the number of times that a terminal voltmeter is observed to rise to a maximum. Another method for making this determination is to sweep between the two frequencies in a given time interval and measure the frequency of the energy pulses occasioned by the voltage rise. This latter principle is the method actually employed.

The Weco Altimeter

This device[11] consists of six major units. A transmitter delivers about 10 watts to a dipole antenna located below one wing of the airplane and is frequency-modulated between 410 and 445 megacycles. This modulation is accomplished at a rate of 60 cycles per second. The energy from the transmitting antenna strikes the ground and is reflected back to the airplane where it is received by an antenna connected to a receiver. The receiver and transmitter are shielded from each other, and the antennas are arranged for minimum coupling; however, a certain amount of energy from the transmitting antenna reaches the receiving antenna and, hence, adds and subtracts to the reflected energy in the manner discussed for the transmission line. The rectified output of the receiver increases and decreases at a frequency which is a function of the distance from the airplane to the ground. Incorporated in the receiver is an electronic-frequency meter. This is really a rate-of-energy pulse counter with an indicating milliameter which is calibrated to read zero to 5000 feet. In order to increase the accuracy of the readings, the scale of the meter extends over a range of 270 degrees. The first 1000 feet are on an expanded portion of the scale, and the smallest division represents 10 feet.

It is necessary to provide certain apparatus for supplying proper voltage and current to the plate and filament of the transmitter tube and to the plates of the receiving tubes, so all equipment of this type is assembled on a single chassis and constitutes the sixth unit of this altimeter.

Performance

The Western Electric absolute radio altimeter is capable of reading distances from 20 to 5000 feet. From 5000 to 15,000 feet, the needle rests against the 5000-foot mark. Above 15,000 feet the readings are somewhat less than 5000 feet. The claimed accuracy within the

[10] W. L. Everitt, "Communication Engineering," McGraw-Hill Book Company, Inc., New York, N. Y., p. 131, 1932.

[11] Lloyd Espenschied and R. C. Newhouse, "A terrain clearance indicator," *Bell Sys. Tech. Jour.*, vol. 18, p. 222; January, 1939.

5000-foot limit is about 10 per cent. Deviation from absolute accuracy is caused by variations in the amount of frequency change, errors in the audio-frequency counter circuit, and errors in the instrument used to read elevation. Since this accuracy is on a percentage basis, its actual value increases for low altitudes and makes the device useful as an instrument landing check.

The weights of the major apparatus units are as follows:

	Pounds
Transmitter	13.9
Receiver	9.56
Power unit	15
Meter	1.25
Two antennas	3.4
Total	43.11

The weight performance is about 116 feet per pound. To this weight must be added that of the transmission lines, mounting rack, and other incidentals common to all aircraft installations of radio apparatus. The total weight installed is about 60 pounds.

One of the biggest objections to this device is the peculiar characteristics of the microwave-oscillator tube filament. The low-voltage filament makes necessary a large power loss. This apparatus takes a total drain of 25.2 amperes from an airplane's 12-volt supply. The usual generator on the airplane has a capacity of 50 amperes; thus, if installed, the altimeter would use one half the power available from one generator. If the tube could be changed to one with a more conventional oscillator filament, this drain would be reduced by about 8 amperes.

Aside from the characteristic listed above, one of the objections to the device lies in its lack of a minimum altitude indicator. A pilot can hardly be expected to eye the meter continually as he flies along the airways, but would do this only for certain maneuvers. If he felt he knew his position accurately, the meter probably would not be consulted. This device, to be useful as an airways warning instrument, should be equipped with a light, buzzer, or other indicator that would warn when elevations of 1000 feet or less have been passed; also, another device to indicate elevations of less than 500 feet might be desirable. This altimeter unfortunately does not include these features. In the experimental models, sensitive current relays were used to provide this warning, but they did not operate successfully because of their susceptibility to vibration.

Standard-Frequency Broadcast Service of National Bureau of Standards*

TWO changes beginning February 1, 1944, are announced in the standard-frequency broadcast service of the National Bureau of Standards. One is the addition of a new radio frequency, 2500 kilocycles per second, at night. The other is omission of the pulse on the 59th second of every minute. The entire service is described here. It comprises the broadcasting of standard frequencies and standard time intervals from the Bureau's radio station WWV near Washington, D. C. The service is continuous at all times day and night, from 10-kilowatt radio transmitters. The services include: (1) standard radio frequencies, (2) standard time intervals accurately synchronized with basic time signals, (3) standard audio frequencies, and (4) standard musical pitch, 440 cycles per second, corresponding to A above middle C.

The standard-frequency broadcast service makes widely available the national standard of frequency, which is of value in scientific and other measurements requring an accurate frequency. Any desired frequency may be measured in terms of any one of the standard frequencies, either audio or radio. This may be done by the aid of harmonics and beats, with one or more auxiliary oscillators.

At least three radio carrier frequencies are on the air at all times, to insure reliable coverage of the United States and other parts of the world. The radio frequencies are:

2.5 megacycles (=2500 kilocycles=2,500,000 cycles) per second, broadcast from 7:00 P.M. to 9:00 A.M. Eastern War Time (2300 to 1300 Greenwich Mean Time).

5 megacycles (=5000 kilocycles=5,000,000 cycles) per second, broadcast continuously day and night.

10 megacycles (=10,000 kilocycles=10,000,000 cycles) per second, broadcast continuously day and night.

15 megacycles (=15,000 kilocycles=15,000,000 cycles) per second, broadcast from 7:00 A.M. to 7:00 P.M., Eastern War Time (1100 to 2300 Greenwich Mean Time).

Two standard audio frequencies, 440 cycles per second and 4000 cycles per second, are broadcast on the radio carrier frequencies of 5, 10, and 15 megacycles.

* Decimal classification: R555. Original manuscript received by the Institute, January 24, 1944.

The audio frequency 440 cycles only is broadcast on 2.5 megacycles. The 440 cycles per second is the standard musical pitch, A above middle C; the 4000 cycles per second is a useful standard audio frequency for laboratory measurements.

In addition there is on all carrier frequencies a pulse of 0.005 second duration which occurs periodically at intervals of precisely 1 second. The pulse consists of 5 cycles, each of 0.001 second duration, and is heard as a faint tick when listening to the broadcast; it provides a useful standard of time interval, for purposes of physical measurements, and may be used as an accurate time signal. On the 59th second of every minute the pulse is omitted.

The two audio frequencies are interrupted precisely on the hour and each five minutes thereafter; after an interval of precisely 1 minute they are resumed. This 1-minute interval is provided in order to give the station announcement and to afford an interval for the checking of radio-frequency measurements free from the presence of the audio frequencies. The announcement is the station call letters (WWV) in telegraphic code (dots and dashes), except at the hour and half hour when a detailed announcement is given by voice.

The accuracy of all the frequencies, radio and audio, as transmitted, is better than 1 part in 10,000,000. Transmission effects in the medium (Doppler effect, etc.) may result in slight fluctuations in the audio frequencies as received at a particular place; the average frequency received is however as accurate as that transmitted. The time interval marked by the pulse every second is accurate to 0.000.01 second. The 1-minute, 4-minute, and 5-minute intervals, synchronized with the seconds pulses and marked by the beginning or ending of the periods when the audio frequencies are off, are accurate to a part in 10,000,000.

The beginnings of the periods when the audio frequencies are off are so synchronized with the basic time service of the United States Naval Observatory that they mark accurately the hour and the successive 5-minute periods.

Of the radio frequencies on the air at a given time, the lowest provides service to short distances, and the highest to great distances. Reliable reception is in general possible at all times throughout the United States and the North Atlantic Ocean, and fair reception throughout the world.

Information on how to receive and utilize the service is given in the Bureau's Letter Circular, "Methods of using standard frequencies broadcast by radio," obtainable on request. The Bureau welcomes reports of difficulties, methods of use, or special applications of the service. Correspondence should be addressed National Bureau of Standards, Washington, D. C.

I.R.E. People

Sylvania Promotes Connor to West Coast Position

George C. Connor, radio field engineer with Sylvania Electric Products, Inc., and active in the radio tube manufacturing industry, has been appointed manager of the California division of his company's equipment tube sales. Mr. Connor's headquarters will be in the Los Angeles office, 555 South Flower Street.

George C. Connor

Mr. Connor has a background of seventeen years in radio engineering, the last ten of them as field engineer in the equipment sales division of Sylvania. He has been an I.R.E. Associate member since 1935. For the past year, Mr. Connor has been working with government radio laboratories on military electronic equipment. Previous to his joining Sylvania, he was associated with the Brunswick Radio Corporation and with the Bremer Tully Manufacturing Company. The West Coast is home territory to Mr. Connor, who was born in Hoquiam, Washington. He has a wide acquaintance with engineers and business men in the west.

W. R. David Named Sales Manager

W. R. David has been named sales manager of broadcast equipment for the transmitter division of the General Electric Company's electronics department. In this capacity, Mr. David will be responsible for the sales of both amplitude- and frequency-modulation broadcast equipment, with headquarters at Schenectady.

A native of Lair, Kentucky, Mr. David earned his B.S. degree in mechanical and electrical engineering at the University of Kentucky (Lexington) in 1919. He was employed by the General Electric Company in July of that year as a student engineer at Schenectady. He has been employed in radio application and sales engineering work since June, 1921.

During this period, Mr. David has had continuous and initimate contact with General Electric radio engineering, research, development, design, as well as radio manufacturing activities and sales work. His proposition, application, and sales engineering experience has included work on spark transmitters for land stations and ships, commercial telegraph and telephone receivers, Alexanderson alternators with all associated apparatus, electronic-tube telegraph and telephone transmitters for land stations and ships, radio direction finders, aircraft radio transmitters and receivers, radio measuring instruments, police radio equipment, radio broadcasting transmitters including all sizes up to 500 kilowatts, and the electron microscope.

W. R. David

Mr. David has been an Associate member of the Institute of Radio Engineers since 1926.

Institute News and Radio Notes

Board of Directors

The 1943 Board of Directors held its final meeting on January 5, 1944. The following attended this meeting: R. A. Heising, treasurer (chairman); H. M. Turner, president-elect; R. A. Hackbusch, vice-president-elect (guest); E. F. Carter, W. L. Everitt, Alfred N. Goldsmith, editor; R. F. Guy (guest); F. B. Llewellyn, B. J. Thompson, and W. B. Cowilich, assistant secretary.

The formal business transacted at this meeting was the approving of the minutes of the December 1, 1943 meeting.

The annual meeting of the Board of Directors took place on January 5, 1944 and was attended by H. M. Turner, president; R. A. Hackbusch, vice-president; W. L. Everitt, Alfred N. Goldsmith, editor; R. F. Guy, R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; B. J. Thompson, H. A. Wheeler, and W. B. Cowilich, assistant secretary.

These applications for membership were approved: for transfer to Senior Member grade, C. E. Atkins, H. H. Brauer, V. N. James, H. B. Martin, C. B. Persons, A. L. Samuel, M. O. Sharpe, and G. R. Town; for admission to Senior Member grade, S. S. Attwood and H. B. Riblet; for transfer to Member grade, J. G. Alverson, J. O. Ashton, Eldridge Buckingham, C. W. Harrison, Jr., and J. H. Hidy; for transfer to Associate grade, 5; for admission to Associate grade, 153; and, for admission to Student grade, 83.

The following officers were reappointed for 1944: R. A. Heising, treasurer; Haraden Pratt, secretary; and Alfred N. Goldsmith, editor.

The five directors appointed for 1944 are S. L. Bailey, E. F. Carter, I. S. Coggeshall, C. B. Jolliffee, and H. J. Reich.

To serve during 1944, the personnel of the following twelve committees were named: Admissions, Awards, Board of Editors, Constitution and Laws, Executive, Investment, Membership, Nominations, Papers, Public Relations, Sections, and Tellers.

The Committee on Registration of Engineers was abolished but its functions were transferred to the I.R.E. Committee on Professional Representation.

H. R. Zeamans was reappointed General Counsel for 1944.

The bank resolutions for the "General Account" and the "Special Account," the latter requiring only the Secretary's signature for withdrawal, were authorized.

Another bank resolution applying to the "Office Account," providing for withdrawal on the signature of the Assistant Secretary and for funds for the payment of current bills, was given approval.

The bank resolution, authorizing the members of the Investment Committee to sign orders on the custodian of the Institute's securities, was also approved.

The actions of the Executive Committee, taken at its meeting on November 30, 1943, were ratified.

On recommendation of the Executive Committee, the proposed 1944 Budget was approved, subject to further study of the indicated trend of increase in rate of disbursements over rate of increase of receipts.

IMPORTANT NOTICE

Members and subscribers are urged to keep their back copies of the Proceedings. **Because of the severe paper limitation, the Proceedings' print order is necessarily restricted. When issues are out of date, they no longer will be available. It is anticipated that after the war there will be a heavy demand for back copies, especially from libraries and foreign sources. These back copies will not be obtainable from the Institute, but it is hoped that members and subscribers can, from time to time and upon request, supply these issues.**

The membership dues of the Institute, which are low in comparison with those of other societies, were given consideration. In view of the upward trend in expenses, the desirability of increasing these dues from the present levels was discussed at length.

The formation of the Dayton (Ohio) Section to include the following counties, recommended by the Executive Committee, was approved:

Counties: Clark, Darke, Greene, Montgomery, Preble, and Warren.

The citations for the Medal of Honor, Morris Liebmann Memorial Prize, and eleven Fellowship awards were accepted.

A report was given relative to the arrangements and budget for the 1944 Winter Technical Meeting.

Arrangements were made for establishing a Committee on Education and appointing the personnel. Approval was given to steps to be taken relative to amending the Bylaws for the purpose of providing for the addition of the Committee on Education to the list of standing committees.

The Engineers' Council for Professional Development was discussed from the standpoint of the Institute's affiliation with that organization.

The resignation of H. A. Wheeler from the chairmanship of the I.R.E. Committee on Professional Recognition, due to the pressure of other work, was accepted with the understanding that Mr. Wheeler would continue to serve as a member of that group. W. C. White, a member of the committee, was appointed chairman.

Treasurer Heising, as chairman of the Office Quarters Committee, reported on the progress being made in the search for a building, suitable for purchase as a permanent home for the Institute.

Editor Goldsmith presented a report relative to the outcome of the Institute's recent appeal, for 1944 paper allotment for the PROCEEDINGS, which had been made to the War Production Board.

A report on the recent activities and trends of the Radio Technical Planning Board and its Panels was made by Secretary Pratt, the Institute's Representative on the named organization.

Executive Committee

At the meeting of the Executive Committee, held on January 4, 1944, the following were present: R. A. Heising, treasurer (chairman); H. M. Turner, president-elect; Alfred N. Goldsmith, editor; Haraden Pratt, Secretary; and W. B. Cowilich, assistant secretary.

These applications for membership were approved and recommended to the Board of Directors for confirming action: for transfer to Senior Member grade, C. E. Atkins, H. H. Brauer, V. N. James, H. B. Martin, C. B. Persons, A. L. Samuel, M. O. Sharpe, and G. R. Town; for admission to Senior Member grade, S. S. Attwood and H. B. Riblet; for transfer to Member grade, J. G. Alverson, J. O. Ashton, Eldridge Buckingham, C. W. Harrison, Jr., and J. H. Hidy; for transfer to Associate grade, 5; for admission to Associate grade, 153; and, for admission to Student grade, 83.

Assistant Secretary Cowilich reported on several office-personnel matters, including the employment of an additional typist, and the extent of the overtime work during December. The decision was made to review salaries of the staff employees.

A request relating to the Engineering Science Management War Training Courses, to be given at Columbia University, was given consideration.

The report on the 1944 budget, prepared by Secretary Pratt, was discussed and recommended to the Board of Directors for conditional approval.

Steps were taken leading to the audit of the Institute's financial records for 1944.

The personnel of the following standing committees, to serve during 1944, were recommended to the Board of Directors for appointment: Admissions, Board of Editors, Investment, Membership, Papers, Public Relations, and Sections.

Editor Goldsmith reported that the proposed personnel for the Papers Procurement Committee would be available in the near future, when it is expected the reorganization of the named committee will have been completed.

It was recommended that the Board of Directors abolish the Committee on Registration of Engineers and transfer its functions to the I.R.E. Committee on Professional Representation.

Editor Goldsmith stated that the Temporary Facsimile Test Standards is in the process of printing and would soon be distributed to the membership.

Approval was granted to the recommendation that the Board of Directors officially establish the Dayton (Ohio) Section.

The proposed budget for the 1944 Winter Technical Meeting was given consideration and proposed for Board acceptance.

High
New York Winter

Chief Engineer of the Federal Communications Commission E. K. Jett addressing the Saturday morning session.

The Convention is opened by Dr. Shackelford flanked by Junior Past-President L. P. Wheeler and President H. M. Turner.

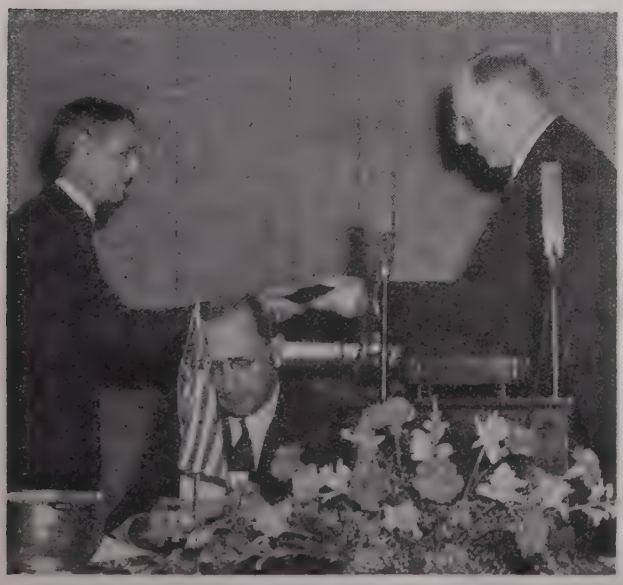
Commander J. J. Raby, U. S. Navy, aviation-radio hero, gives an inspiring address at the banquet.

W. L. Barrow receives from the President the Morris Liebmann Memorial Prize.

Lights Technical Meeting

Part of the Sections Committee hard at work. (Left to right, facing camera and table) Dr. W. L. Everitt, Director; A. B. Bronwell, Chicago Section; E. S. Heiser, Twin Cities; W. M. Smith, Connecticut Valley; R. S. Ould, Washington; H. D. Johnson, Emporium; (partly hidden) F. C. Everett, Cleveland; C. F. Dougherty, Atlanta; G. T. Royden, Admissions Committee; Past-President L. P. Wheeler; J. A. Fitch, Washington; LeRoy Fiedler, Buffalo-Niagara. (Back to camera, foreground to right background) Beverly Dudley, Chicago; D. J. Tucker, Dallas; B. R. Teare, Pittsburgh; C. R. Town, Rochester; J. V. Wilcox, Kansas City; (almost hidden) W. P. West, Philadelphia; John Miller, New York; (head only showing) E. E. Alden, Indianapolis; A. N. Curtiss, Indianapolis. (Back row, left to right) J. C. R. Punchard, Montreal; L. T. Bird, Montreal; R. G. Anthes, Toronto; E. O. Swan, Toronto; H. B. Richmond, Boston; R. E. Hopkins, Los Angeles.

W. R. G. Baker summarizes the organization and work of Radio Technical Planning Board.



Haraden Pratt receives the I.R.E. 1943 Medal of Honor from President Turner.

Major General Roger B. Colton, of the Signal Corps, at the banquet. The previous night he addressed A.I.E.E. and I.R.E. in joint session.

1944 Winter Technical Meeting

The Winter Technical Meeting was held on January 28 and 29 in New York City, with headquarters at the Hotel Commodore. A total of 1704 members and guests were registered. The average daily registration of 852 is the highest in the history of such Institute meetings.

During the two days there were four technical sessions at which 22 papers were delivered.

Dual sessions were conducted during the morning and afternoon of the first day and received the approval of those attending. The large attendance at all sessions indicated the careful arrangement of the papers, particularly at the dual sessions.

These papers were presented at the meeting:

Friday, January 28

"Electronic Tin Fusion," by H. C. Humphrey, Westinghouse Electric and Manufacturing Co.

"The Amplidyne System of Control," by E. F. W. Alexanderson, K. K. Bowman, and M. A. Edwards, General Electric Company.

"Joint Army and Navy Tube Standardization Program," by Lieutenant C. W. Martel, United States Army, and J. W. Greer, United States Navy.

"A New Studio-to-Transmitter Antenna," by M. W. Scheldorf, General Electric Company.

"Orthicon Cameras in Television Studio Work," by H. R. Lubcke, Don Lee Broadcasting System.

"The Modification of Noise by Certain Nonlinear Devices," by D. O. North, Radio Corporation of America.

"Some Experiments Relating to the Statistical Theory of Noise," by C. M. Burrill, Radio Corporation of America

"Intermittent Behavior in Oscillators," by W. A. Edson, Bell Telephone Laboratories, Inc.

"The Limitations Imposed by Quantum Theory on Resonator Control of Electrons," by L. P. Smith, Cornell University (temporarily RCA Consultant).

"The Piston Attenuator," by H. A. Wheeler, Hazeltine Electronics Corporation.

"Equivalent Circuits for Discontinuities in Transmission Lines," by J. R. Whinnery and H. W. Jamieson, General Electric Company.

"Transmission-Line Analogies of Plane Electromagnetic Waves," by A. B. Bronwell, Northwestern University.

Three Papers, viz.:

"Equivalent Circuit of the Field Equations of Maxwell," by Gabriel Kron.

"A New Approach to the Solution of High-Frequency Field Problems," by J. R. Whinnery and Simon Ramo.

"Alternating-Current Network-Analyzer Studies of Electromagnetic-Cavity Resonators," by J. R. Whinnery, C. Concordia, W. Ridgway, and Gabriel Kron.

Presented by J. F. McAllister, J., General Electric Co.

Symposium: "The Work of the Radio Technical Planning Board."

W. R. G. Baker, Chairman of the Radio Technical Planning Board.

Alfred N. Goldsmith, Chairman of Panel 1—*Spectrum Utilization*

C. B. Holliffe, Chairman of Panel 2—*Frequency Allocation*

R. M. Wise, Chairman of Panel 3—*High-Frequency Generation*

H. S. Frazier, Chairman of Panel 4—*Standard Broadcasting*

C. M. Jansky, Jr., Vice-Chairman of Panel 5—*Very-High-Frequency Broadcasting*

D. B. Smith, Chairman of Panel 6—*Television*

J. V. L. Hogan, Chairman of Panel 7—*Facsimile*

Haraden Pratt, Chairman of Panel 8—*Radio Communication*

E. W. Engstrom, Chairman of Panel 9—*Relay Systems*

W. P. Hilliard, Chairman of Panel 10—*Radio Range, Direction, and Recognition*

D. W. Rentzel, Chairman of Panel 11—*Aeronautical Radio*

C. V. Aggers, Chairman of Panel 12—*Industrial, Scientific, and Medical Equipment*

D. E. Noble, Chairman of Panel 13—*Portable, Mobile, and Frequency Service Communications*

Saturday, January 29

"Design Technique versus Service Requirements," by I. W. Stanton, Radio Corporation of America.

"Radio in Service of Home and Nation," by Arthur Stringer, National Association of Broadcasters.

Symposium: "Engineering Work of the Federal Communications Commission."

"General Introduction," by E. K. Jett, Chief Engineer of the Federal Communications Commission and Chief of Several Engineering Divisions.

"Timely Broadcast Matters," by G. P. Adair, Assistant Chief Engineer and Chief of the Broadcast Division of the FCC Engineering Department.

"Police, Aviation, and Maritime Services," by W. N. Krebs, Chief of the Safety and Special Services Division of the FCC Engineering Department.

"International Point-to-Point and Allocation Problems," by P. F. Siling, Chief of the International Division of the FCC Engineering Department.

"Radio Progress in Canada," by R. A. Hackbusch, Vice President and Managing Director, Stromberg-Carlson Company, Ltd.

"Peace, War, and Future Application of Radio in China," by T. M. Liang, Chinese Supply Mission.

"Standardization of Service Equipment," by Commander A. B. Chamberlain, United States Navy.

The Chairmen at the technical sessions were H. M. Turner, president-elect; L. P. Wheeler, junior past president; F. S. Barton, junior past vice-president; Haraden Pratt, secretary; R. A. Hackbusch, vice-president-elect; and Lloyd Espenschied, chairman-elect of New York Section.

Exhibits of the captured enemy radio equipment and the communications equipment standards for the Army, Navy and Air Corps were centers of considerable interest.

The banquet held on Friday evening, January 28, at the Hotel Commodore, was attended by 808 members and guests, the largest group ever to assemble at an Institute banquet. The program, under the direction of George Lewis as Master of Ceremonies, included presentation of awards; the annual address of the retiring president, by L. P. Wheeler; and the speaker of the evening, Commander J. J. Raby, United States Navy.

The awards, presented by President Turner, are given below with the names of the recipients and the citation in each case:

Medal of Honor for 1944

Haraden Pratt. In recognition of his engineering contributions to the development of radio, of his work in the extension of communication facilities to distant lands, and of his constructive leadership in Institute affairs.

Morris Liebmann Memorial Prize for 1943

Wilmer L. Barrow. For his theoretical and experimental investigations of ultra-high-frequency propagation in wave guides and radiation from horns, and the application of these principles to engineering practice.

Fellowships

Stuart L. Bailey. For pioneering accomplishment in the application of radio engineering principles to the solution of technical problems in broadcasting.

Charles R. Burrows. For his contributions in the field of radio wave propagation, particularly for his investigations of propagation along the ground at ultra-high frequencies.

Murray C. Crosby. For his contributions to the development of high-frequency radio communications, including a careful study of frequency modulation.

Harry Diamond. For his contributions to the development and application of radio aids in air navigation and meteorology.

Carl B. Feldman. For his investigations of the characteristics of radio waves and his developments in antennas and receiving systems.

Keith Henney. In recognition of his accomplishments in obtaining publication of the technical information essential to the radio engineer.

Dwight O. North. For his contributions to the knowledge of the fundamentals of vacuum-tube performance, especially with regard to fluctuation phenomena and the effects of electron transit time.

Kenneth A. Norton. In recognition of his work in applying his conclusions from the theory of radio wave propagation to the problems of frequency allocation.

Stuart W. Seeley. For the development of practical apparatus for the radio industry, particularly in the fields of television and high-frequency broadcast reception.

Donald B. Sinclair. For the development and application of various types of networks for high-frequency measurement of impedance.

Leo C. Young. In recognition of his pioneer work on the causes of the vagaries of short-wave propagation and for his invaluable contributions to the efficient operation of an outstanding research laboratory.

Messrs. Pratt and Barrow, recipients of the two highest awards of the Institute, delivered timely speeches.

Those called upon to speak were Major General R. B. Colton, Signal Corps, United States Army; E. K. Jett, chief engineer, Federal Communications Commission; W. R. G. Baker, chairman, Radio Technical Planning Board; F. S. Barton, junior past vice-president of the Institute; R. A. Hackbusch, vice-president-elect of the Institute; R. H. Marriott, first president of the Institute; and B. E. Shackelford and Austin Bailey, chairman and vice-chairman of the Winter Technical Meeting, respectively.

A large number of members attended the annual meeting of the Sections Committee, which was held on Thursday, January 27, at the Hotel Commodore. Many of the Sections were represented and the several matters on the agenda were discussed at length. Among the subjects given consideration were Section reports, increasing the scope of Institute activities, the need for increasing membership dues, and the desirability of purchasing a permanent home for the Institute.

The joint A.I.E.E.-I.R.E. session on the evening of January 27, at the Engineering Societies Building in New York City, was well attended. The paper of the evening, "Enemy Army Communications Equipment," was presented by Major General R. B. Colton, Signal Corps, United States Army, and followed by an inspection of exhibited captured apparatus.

The annual meeting of the Institute, with President Turner presiding, was called to order at approximately 11 o'clock on Friday morning, January 28. The meeting was devoted to amending the Institute's charter, on which the notice was mailed during December 1943 to all members.

It should be noted that preprint copies of the papers, given at the Winter Technical Meeting, were not made and therefore are not available from the Institute. It is the intention to have as many of these papers as possible published in the forthcoming issues of the PROCEEDINGS, but no assurance can be given that all of them will be published.

The general committee for the Winter Technical Meeting consisted of B. E. Shackelford, chairman; Austin Bailey, vice-chairman; I. S. Coggeshall, E. J. Content, W. B. Cowilich, H. F. Dart, G. A. Downsbrough, Alfred N. Goldsmith, F. A. Gunther, George Lewis, F. B. Llewellyn, J. R. Poppele, Haraden Pratt, H. A. Wheeler, and L. J. Woods.

Correspondence

A Note on Frequency-Modulation Terminology

The fundamental theory of frequency modulation is simple and straightforward. With the exception of sideband analysis involving Bessel functions the mathematics required to present frequency modulation is on a level covered by any undergraduate course in engineering. Still the frequency-modulation theory is quite an obstacle to the average student or engineer, who often seems to get more confused regarding the fundamental principles the more articles and books he reads on the subject. This state of affairs is probably due to the rapid developments in the frequency-modulation field, which has resulted in a-rather vague terminology and few generally adopted definitions.

The confusion about fundamental concepts is present in the term modulation itself. This term is used in two different senses:

1. *as a characteristic property of a wave,*
2. *as the process of, or method for, changing a pure sine wave into a wave of that property.*

The first sense is more fundamental than the second, but the present modulation terminology is largely based on the latter. In this connection it is natural to refer to the excellent work done by The Institute of Radio Engineers in their Standards of 1938, although it is hardly to be expected that five years of rapid development in the field have left these standards untouched by obsolescence. In the Standards (1T24 and 1T25) the difficulties just mentioned are avoided by reserving the term "modulation" for the second sense only and using the term "modulated wave" to express the first sense. This is logical and satisfactory but somewhat contrary to general usage. We do not think this is too serious, but everyone using the word "modulation" should be aware of its ambiguity and make clear what he means.

Another cause of much unnecessary controversy is that in general no clear distinction is made between a single "modulated wave" and a "*transmission* of modulated waves." Whenever the collective properties of all possible modulated waves in a certain transmission channel, or the waves produced by a particular process of modulation, are discussed, the term "modulated wave" is inadequate. The word "transmission" seems the collective term most suitable for this purpose.

An Approach to the Theory

Attacking the subject from the fundamental aspect we have a sine wave

$$e = E_m \sin \phi(t), \tag{1}$$

where e is the instantaneous value, E_m the real amplitude, and $\phi(t)$ the instantaneous (electric) angle, which may be written

$$\phi(t) = \phi_0 + \phi_1 t + \phi_2 t^2 + \phi_3 t^3 + \cdots. \tag{2}$$

The instantaneous frequency f_t and the instantaneous angular velocity $\omega_t = 2\pi f_t$ are defined by the relation

$$\omega_t = d/dt\,(\phi)t, \tag{3}$$

which gives

$$\phi(t) = \phi_0 + \int_0^t \omega_t\, dt. \tag{4}$$

If we, for instance, want to use the wave to carry intelligence, we may control E_m or $\phi(t)$ or both. *Only two fundamental, independent types of modulated waves are possible.* All other types are necessarily modifications or combinations of the two. The first type is known as an *amplitude-modulated wave*, the other type is best described as a *frequency-modulated wave* (or angle-modulated wave). The instantaneous angle $\phi(t)$ may be expressed in another parameter than frequency, but the frequency seems to be the most important parameter. Note that in the above discussion no limitation is imposed on the fashion in which the frequency may vary, no relation given between the signal voltage and the frequency variation produced. The concepts introduced are, and should be, as general as possible.

With reference to the above discussion the following tentative definitions are formulated:

Definition 1. *An amplitude-modulated sinusoidal wave is a wave whose amplitude varies as a function of the instantaneous value of another wave (for instance, a signal wave).*

Definition 2. *A frequency-modulated sinusoidal wave is a wave whose instantaneous frequency varies as a function of the instantaneous value of another wave (for instance, a signal wave).*

These proposed definitions follow closely the I.R.E. definition of a modulated wave (1T25) but are considerably more general than the I.R.E. definitions of the same terms (1T26, 1T28), which, for instance, do not account for all kinds of spurious modulations often encountered in practice. Further, those definitions introduce a strict relation between wave and signal, appropriate when defining a *method of modulation* but hardly justified in a general definition of a *modulated wave.*

A frequency-modulated wave described by Definition 2 may be written, with full generality,

$$e = E_m \sin \phi(t) = E_m \sin \left(\int_0^t \omega_t\, dt + \phi_0 \right) \tag{5}$$

where ϕ_0 is the initial phase angle or initial phase. The instantaneous angular velocity of the voltage phasor[1] may, in a special case of particular interest, be of the form

$$\omega_t = \omega + \Delta\omega \cos \Omega t, \tag{6}$$

[1] The term "phasor" is preferred for vector, which thus could be reserved for exclusive use in the vector analysis. The term "phasor" is used by the American Institute of Electrical Engineers. It may be of interest to mention that the European engineers have been using for many years a term, which in translation would be called "pointer" (referring to the pointer or hand of a meter or a clock).

where Ω is the angular velocity of the frequency deviation.

This gives

$$e = E_m \sin\left[\int^t (\omega + \Delta\omega \cos \Omega t)dt + \phi_0\right],$$

or

$$e = E_m \sin\,[\omega t + m_f \sin \Omega t + \phi_0'], \quad (7)$$

where $m_f = \Delta\omega/\Omega = \Delta f/F$ = modulation index.

The restrictions imposed so far by using a sine wave are not serious, as all periodic waves can be built up from sine-wave components. A single frequency-modulated wave is defined by a set of values E_m, ω, Ω, m_f, and ϕ_0', which do not reveal anything about the source or the method used for its generation.

However, a *transmission* utilizing frequency-modulated waves will generally contain waves with a wide range of modulation indexes, and the nature of the transmission will depend very much on how the modulation index varies with signal amplitude and frequency. Fundamentally, there are three cases:

(a) The frequency deviation Δf is proportional to the signal amplitude and independent of the signal frequency,

(b) the modulation index m_f is proportional to the signal amplitude and independent of the signal frequency,

(c) the modulation index m_f is proportional to the signal amplitude but is any other function of the signal frequency than in (a) and (b).

Thus, although there are logically only two types of modulated waves, amplitude and frequency modulation, transmissions utilizing frequency modulation require further classification. As it is largely a question of the audio-frequency (or signal-frequency) response of the modulating equipment, a similar classification of amplitude-modulated waves is quite possible, although there is no need for it.

What about phase modulation? To see that question in its proper light, let us consider *methods* of producing frequency-modulated waves. If the output of the microphone or television camera linearly controls ω, a transmission of type (a) is obtained. Equation (7) illustrates the produced waves with Ω representing the frequency of the modulating signal. Another well-known method of producing frequency-modulated waves is to add to the electric angle a systematically controlled phase angle $\theta(t)$. (The initial phase angle ϕ_0 (equation (2), (4) and (5)) is by definition a constant and can not serve as a control variable.)

Keeping ω constant and adding the phase quantity $\theta(t)$, we obtain

$$e = E_m \sin\,[\omega t + \phi_0 + \theta(t)], \quad (8)$$

so that for an assumed modulation

$$\theta(t) = \theta + \Delta\theta \sin \Omega t, \quad (9)$$

$$e = E_m \sin\,[\omega t + m_f \sin \Omega t + \phi_0'], \quad (10)$$

where $m_f = \Delta\theta$.

This expression for a frequency-modulated wave is identical with (7), but a *transmission* using this kind of modulator will not be the same as produced by a reactance-tube modulator, because m_f depends on the signal frequency in a different manner: if $\Delta\theta$ is independent of the signal frequency, we shall obtain a transmission of type (b) above.

Suggested Technical Terms

It is tempting to refer to transmissions of type (a) as "frequency modulation" and of type (b) as "phase modulation" and it is often done.[2] Let us consider this critically. First, would the word "modulation" introduce any ambiguity when used to classify transmissions? In our opinion, this is not the case, since the properties of a *transmission* of modulated wave is very intimately connected with the equipment used to produce it. It is very serious, however, that we would use the word "frequency-modulated," classifying modulated waves, and the words "frequency modulation," classifying the transmissions, with different meanings. In the first case there are no restrictions imposed on the relation between "signal wave" and frequency deviation, in the second there is a very narrow restriction, as expressed in the description of type (a) above. Similarly, in type (b) there is a restriction on the relation between "signal wave" and phase deviation, that would not be indicated by the term "phase modulation." We must use terms that clearly express these restrictions. Now, according to common parlance, a transducer where the ratio of output to input is largely independent of frequency is said to have a flat response. This suggests the terms "flat frequency modulation" and "flat phase modulation" for transmissions of type (a) and (b), respectively. In the last term it might seem desirable to discard the controversial expression "phase modulation" altogether, but we have not been able to find another name as brief and significant as "flat phase modulation."

It is well known that present frequency-modulation broadcast stations do not use transmissions of either type (a) or (b), but rather a compromise, or modification, of the two. This compromise is to its nature described by the high-tone emphasis curve recommended by the Federal Communications Commission and actually used in some instances in the amplitude-modulation field before the advent of commercial frequency modulation. As this type of transmission is adopted as a standard, "standard frequency modulation" (or standard broadcast frequency modulation) seems the logical name for it. The definition of the terms introduced above are then as follows:

Definition 3. A transmission of frequency-modulated waves is said to use *flat frequency modulation*, FFM, if the amplitude of the frequency deviation from the nominal (carrier) frequency is proportional to the amplitude and independent of the frequency of the modulating wave (for instance a signal wave).

Definition 4. A transmission of frequency-modulated waves is said to use *flat phase modulation* FPM, if the amplitude of the phase deviation from the average phase is proportional to the amplitude and independent of the frequency of the modulating wave (for instance a signal wave).

Definition 5. A transmission of frequency-modulated waves is said to use *standard frequency modulation*, SFM, if the amplitude of the frequency deviation from the nominal (carrier) frequency is proportional to the amplitude of the signal to be transmitted but varies with the signal frequency according to the standard accentuation curve.

The various terms introduced above may be grouped in Table I:

TABLE I

The Entire Art		Type of Transmission
FM	FFM	flat frequency modulation
	FPM	flat phase modulation
	SFM	standard frequency modulation

The SFM transmission may be considered as a combination of FFM at low modulation frequencies and FPM at high modulation frequencies, or SFM may be thought of as FFM transmission with the signal emphasized above a given crossover frequency. This brings up for consideration the term *emphasizer*, and for the receiving side the corresponding term *de-emphasizer*. Confusion often occurs when these terms are mixed with the differentiating or integrating action employed in practice to make possible the use of any type of equipment for any type of transmission. The devices producing the differentiating or integrating action are, if ideal, differentiators and integrators, and these terms may be used to distinguish sharply against networks employed for emphasis or de-emphasis from a given crossover frequency. The simultaneous appearance of differentiation, integration, emphasis and de-emphasis in a complete system is illustrated by Table II.

TABLE II

FM Transmitters	FM Transmission	FM Receivers
Reactance-tube modulator	FFM	Receiver for FFM
Integrator plus phase modulator	FFM	(Receiver for FPM plus differentiator)
Phase modulator	FPM	Receiver for FPM
Differentiator plus reactance-tube modulator	FPM	(Receiver for FFM plus integrator)
Emphasizer plus reactance-tube modulator	SFM	Receiver for SFM
	SFM	(Receiver for FFM plus de-emphasizer)
Integrator plus emphasizer plus phase modulator	SFM	(Receiver for FPM plus differentiator plus de-emphasizer)

Terms such as "reactance-tube modulator" and "phase modulator" are used only to

[2] See H. Stockman, "Superheterodyne converter terminology," *Electronics*, vol. 16, pp. 144–331; November, 1943. The fundamental idea of classifying transmissions was outlined in a footnote, but subsequent discussion has resulted in the new terms given above.

give an idea of one possible arrangement on the transmitting side.

The thoughts here presented are intended to provide a common foundation for discussion. Much controversy and confusion is caused when a new rapidly developing field borrows words from common language without defining explicitly the new concepts they shall cover. The basic definitions must fulfill a number of severe requirements; they must be *fundamental*, *logical*, *clear*, and must satisfy *practical* needs, in school as well as in the field.

We express the hope that these lines may stimulate other workers to clarify the definitions and improve the terminology.

HARRY STOCKMAN
Cruft Laboratory

GUNNAR HOK
Radio Research Laboratory

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Cambridge, Massachusetts

Books

Moderne Mehrgitter—Elektronenröhren (Modern Multigrid Electron Tubes), by M. J. O. Strutt. Second, enlarged and improved edition.

Published (1940) by Julius Springer, Berlin, Germany. Now available in Lithoprint from Edwards Brothers, Inc., Ann Arbor, Mich. 278 pages + 5-page index +VIII pages. 242 figures. $8\frac{3}{4}\times5\frac{3}{4}$ inches. Price, $5.00.

This book is a new edition which includes both parts of the previous two-volume work under the same title. The earlier books were published very soon after their manuscripts had been completed. In the present edition, Dr. Strutt has made a number of improvements and the presentation was revised so as to include new material. The first of the two main parts covers the construction, operation, and characteristics of multigrid tubes, and is divided into a section on high-frequency amplification, one on mixers, and one on audio-frequency power amplifiers. The second main part treats the fundamental physical principles involved in such tubes and has a section on tube behavior under quasi-stationary conditions and one on the behavior in the short-wave region. Thus, although the first part is of direct interest chiefly to the user of tubes, the second part is of particular concern to vacuum-tube design engineers.

The book presents a comprehensive picture of multigrid receiving tube practice as it stood at the beginning of the war. The author was well qualified to write such a book, having previously published much of the material as original work during his many years with the Philips organization. Although a number of the details will become somewhat outmoded by war developments, especially in the ultra-high-frequency field, much of the book is sufficiently fundamental to retain its usefulness. It should be noted that, in accord with the title, diode and triode tubes receive practically no attention, probably on the assumption that these are adequately treated in standard texts. This is indeed true, as far as fundamentals are concerned; however, the omission limits the reference value of the present book since it contains no material on oscillators, rectifiers, diode detectors, and radio-frequency power amplifiers. On the other hand, the book emphasizes details of comparatively recent receiving-tube developments, such as converter tubes, and treats high-frequency tube admittances and fluctuation noise from a modern point of view; this distinguishes the book from the less specialized standard texts and makes it of considerably greater value to radio receiving tube and radio receiver engineers.

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Princeton, N. J.

Short Wave Wireless Communication, Fourth Edition, by A. W. Ladner and C. R. Stoner

Published (1943) by John Wiley and Sons, Inc., 601 West 26 Street, New York 1, N. Y. 568 pages + 5-page index + xiv pages. 342 figures. $6\times8\frac{1}{2}$ inches. Price, $6.00.

This book covers in a general way the principles and practices of short- and ultra-short-wave radio communications. Certain sections of the book describe equipment and practices followed abroad and particularly in England. The fourth edition contains much of the material contained in the previous editions. It has been revised and brought up to date, insofar as the authors were permitted to do so under wartime restrictions, by the addition of new material. The chapters on propagation, transmission lines, and antennas cover these subjects in a clear and concise manner.

The book is largely nonmathematical. The mathematics which appear in the book are not involved and should be readily followed by those who would be interested in the subject matter of this book.

The book is well written, clear, easily understood, and generously illustrated. There are included 151 selected references which are grouped at the end of the chapters and pertain to the subject covered by the proceeding chapter. The references are useful to those readers who wish to investigate and study a subject further.

This book would be useful to executives and engineers in radio communications to provide them with general knowledge of the practices and problems encountered in the short wave radio field. It also would be a useful reference book to students in communications courses to give them an outline of the problems and practices in the practical application of the more theoretical principles which they are studying and in particular to the practices followed abroad.

CARL E. SCHOLZ
Mackay Radio and Telegraph Company
67 Broad Street
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Principles of Aeronautical Radio Engineering, by P. C. Sandretto

Published (1943) by McGraw-Hill Book Company, 330 West 42 Street, New York 18, N. Y. 406 pages+8-page index+xiii pages. 223 figures. $6\frac{1}{2}\times9\frac{1}{4}$ inches. Price, $3.50.

This is a survey of the special problems encountered in the extension of radio engineering into the aeronautical field. It assumes a general knowledge of radio engineering principles and is essentially a practical book. The author has been objective in his approach to the problems and perhaps unduly modest in the discussion of them.

Radio ranges, direction finders, markers, instrument landing systems, absolute altieters, and communication systems form the main divisions of the book. The discussions are well balanced so far as is possible within the limits of present restrictions. It is, of course, impossible to treat fully each of the subjects listed above within the covers of a book of this size. However, there is much practical information for both the designing and the operating engineer.

At the end of each chapter, there is a short bibliography, and at the end of the book an appendix, "Mechanical Requirements for Aircraft Radio Equipment," as specified by the airline companies.

Additional editing of the text would have been helpful, and in the interest of eliminating an occasional awkward sentence. The frequent statement that a matter referred to would be "discussed later" without the specific reference is at times annoying. These minor details, however, detract little from the value of Colonel Sandretto's contribution.

B. E. SHACKELFORD
Radio Corporation of America
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New York 19, N. Y.

Graphical Constructions for Vacuum Tube Circuits, by Albert Preisman

Published (1943) by McGraw-Hill Book Co., 330 W. 42 St., New York 18, N. Y. 234 pages + 3-page index + x pages. 125 figures. $5\frac{1}{2}\times8\frac{1}{2}$ inches. Price, $2.75.

The author defines the scope of this new volume as covering graphical constructions for solutions to problems involving nonlinear circuits, particularly those involving vacuum tubes. This subject is covered quite completely. The book will be of value to designers of equipment using vacuum tubes and also to designers of tubes themselves. The chapter on balanced amplifiers should be particularly useful. The chapter on reactive loads gives a powerful method for attacking problems involving reactive loads. For practical purposes such problems are most quickly solved experimentally, but nevertheless it occasionally is useful to have another method of solution, even though quite laborious. The information is presented clearly and with considerable detail so that an extensive background is not needed to follow the reasoning. In discussing the op-

timum load resistance for class A amplifiers, the author is apparently not aware of Warner and Loughren's excellent paper in the December, 1926, PROCEEDINGS of the I.R.E., with the result that he fails to bring out the fact that the optimum load resistance is twice the tube-plate resistance only if the plate-dissipation rating of the tube is not exceeded. This section of the book is somewhat vague. On the whole, this volume should be valuable to engineers in the electronic field.

E. E. SPITZER
Radio Corporation of America
Lancaster, Pa.

Electric Circuits, by the Department of Electrical Engineering of the Massachusetts Institute of Technology.

Published (1940) by John Wiley and Sons, Inc., 601 West 26 Street, New York, 1, N. Y. 767 pages + 14-page index + xxxiii pages. 420 figures. $6\frac{1}{2} \times 9\frac{1}{4}$ inches. Price, $6.50

This is a new printing of one of the series of books on different branches of electrical engineering written, in collaboration, by members of the staff of the Department of Electrical Engineering at the Massachusetts Institute of Technology. This series is intended to provide "a basic course covering subjects of fundamental importance for all students of electrical engineering, regardless of their ultimate specialty." In its scope the present work includes cases where radiation in the form of electromagnetic waves plays a small or minor role, that is, briefly, circuits of lumped rather than distributed constants.

After an extensive introductory section on fundamental circuit parameters, their calculation and representation in concrete form, there begins the main part of the book, that is, the chapters on circuit analysis.

Here the treatment progresses by steps in logical order from the case of simple resistance networks acted on by constant applied electromotive forces to transients in circuits including inductance and capacitance also, acted on by suddenly applied constant voltage. A chapter on steady-state values in alternating-current circuits leads to the case of transients occurring during switching operations, and so on to coupled circuits and multibranch networks. Chapters are included also on polyphase currents, theory of symmetrical components, electromechanically coupled circuits, and transients in nonlinear circuits.

The whole development leaves nothing to be desired in the way of clearness of statement and attention to detail. Especial attention is directed in all cases to emphasis and reiteration of the underlying assumptions and limitations. The teacher will find the book a veritable mine of interesting and suggestive material. The many problems are devised to furnish interesting and pertinent illustrations of the subject matter of the text. Especially readable and illuminating are the introductory paragraphs at the beginning of each section. The electrical engineer will find the book a valuable reference source. For the average undergraduate student, however, the treatment is so advanced and the ground covered is so extended that the use of the book should follow an orientation course of a more elementary nature.

FREDERICK W. GROVER
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Radio Materiel Guide, by Francis E. Almstead and F. R. L. Tuthill

Published (1943) by McGraw-Hill Book Co., 330 W. 42 Street, New York 18, N. Y. 130 pages + 6-page index. 155 figures. $5\frac{1}{2} + 7\frac{1}{2}$ inches. Price, $2.00.

This book was planned as the classroom text for a short course in radio theory and equipment for military applications. It is hard to see how this text can be stretched to cover the 16-week training period stated, since many subjects are covered with the conciseness of a dictionary, by a sentence or a paragraph at the most. It will at least give the trainee an idea of how the commoner pieces of apparatus appear and what they are for, if not how they function.

RALPH R. BATCHER
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Fundamental Radio Experiments, by Robert C. Higgy

Published (1943) by John Wiley and Sons, Inc., 601 W. 26 St., New York, 1, N. Y. 91 pages + 3-page index + vi pages. 71 figures. $6 \times 8\frac{3}{4}$ inches. Price, $1.50.

This book shows a good selection of types of apparatus to be experimented upon, is well laid out, has good diagrams, and the first few experiments are good.

On the other hand, the advanced experiments are much too short. The author attempts to cover too much ground in a small text. It is not suitable for college grade work, although it might find application in short-term qualitative courses, such as some ESMWT.

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Contributors

MANFRED BROTHERTON

Manfred Brotherton was graduated from the University of London, King's College, in 1921, after he had seen service with the British Army during World War I. Subsequently, he worked with Professor O. W. Richardson in thermionic research and was awarded the doctorate of philosophy in 1924. After joining the apparatus development department of Bell Telephone Laboratories in 1927, Mr. Brotherton spent several years in the development of filters and equalizers and he is now engaged in the development of paper condensers.

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Frederick B. Llewellyn (A'23-F'38) was born September 16, 1897, in New Orleans, Louisiana. Between 1915 and 1922 he spent a

FREDERICK B. LLEWELLYN

total of three years as a radio operator with the United States Navy and on ships of the merchant marine. In 1922 he was graduated from Stevens Institute of Technology with the degree of Mechanical Engineer, and in 1928 received the degree of Doctor of Philosophy in Physics from Columbia University. Joining the engineering department of the Western Electric Company in 1923, he was transferred to the Bell Telephone Laboratories when that company was formed in 1925 and has remained with them ever since. He has been primarily concerned with radio and circuit research which has extended to the analysis of the electronic behavior of vacuum tubes at high frequencies. Several papers by Dr. Llewellyn have appeared in the PROCEEDINGS and in 1936 he was awarded the Morris Liebmann prize for work on constant-frequency oscillators and on vacuum-tube electronics at high frequencies.

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Dah-You Maa* was born on March 1, 1915, in Peiping (then Peking) China. He was graduated from the National University of Peking (Peiping) in 1936 with a degree of

DAH-YOU MAA

B.Sc. in physics. He held a Tsinghua fellowship, won by competitive examination, from August, 1937, to January, 1940, to pursue graduate work in the United States. Dr. Maa spent one year at the University of California in Los Angeles and the following two at Harvard where he received a Whiting fellowship during 1939–1940. He received the degrees of M.A. in 1939 and Ph.D. in physics in 1940 from Harvard. From 1940 to 1942 he was an assistant professor and later professor in electrical engineering at the National Tsinghua University (Kunming, China). Dr. Maa has published a few papers on acoustics and is still working in that field.

* Paper published in July, 1943, issue of the PROCEEDINGS.

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L. C. PETERSON

Stuart L. Parsons was born in Gobles, Michigan, on September 6, 1912. He received the B.S. degree in physics in 1938 and the M.S. degree in physics in 1939 from the University of Michigan. He was a research assistant in the department of engineering research in Michigan, where he worked under Drs. Sawyer and Vincent on the development of apparatus and techniques for spectrochemical routine control laboratories. Mr. Parsons joined the research laboratory of Sylvania Electric Products, Inc., in September, 1939, where he has worked on spectroscopic problems and is at present in charge of the optics section.

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L. C. Peterson (A'32) was born in Varberg, Sweden. He studied at Chalmers Technical University in Gothenberg and took further courses at the Technical Universities in Berlin and Dresden in Germany. After finishing these studies, Mr. Peterson took the test course at the General Electric Company in Schenectady. A year later he became a member of the development and research department of the American Telephone and Telegraph Company. In 1931 he transferred to the Bell Telephone Laboratories as a member of the Technical Staff. Here his work has been largely concerned with the analysis of circuits and with vacuum-tube performance at radio frequencies.

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Peter C. Sandretto (A'30-M'40-SM'43) was born April 14, 1907, at Pont Canavese, Italy. He received the B.S. degree in electrical engineering from Purdue University in 1930 and the E.E. degree in 1938. From 1925 to 1930 he was a broadcast radio operator; from 1930 to 1932 a member of the technical staff of the aircraft radio group of the Bell Telephone Laboratories; from 1932 to 1938 a communications engineer for United Air Lines Transport Corporation, and served as superintendent of the communications laboratory for United Air Lines until 1942, when he entered military service. He was assistant chief of the radar division, Directorate of Communications, until 1943, at which time he served as signals liaison officer at the Air Ministry in London. Lieutenant

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Colonel Sandretto is at present director of the analytical department of the First Proving Ground Electronics Unit of the Air Force. Since 1942 he has been on the Papers and Papers Procurement Committees of the I.R.E. He is a member of Eta Kappa Nu.

THE INSTITUTE OF RADIO ENGINEERS

INCORPORATED

SECTION MEETINGS

ATLANTA	CHICAGO	CLEVELAND	DETROIT	LOS ANGELES
March 17	March 17	March 23	March 17	March 21
NEW YORK	**PHILADELPHIA**	**PITTSBURGH**	**PORTLAND**	**WASHINGTON**
April 5	April 6	April 10	April 10	April 10

SECTIONS

ATLANTA—Chairman, C. F. Daugherty; Secretary, Ivan Miles, 554—14 St., N. W., Atlanta, Ga.

BALTIMORE—Chairman, G. J. Gross; Secretary, A. D. Williams, Bendix Radio Corp., E. Joppa Rd., Towson, Md.

BOSTON—Chairman, R. F. Field; Secretary, Corwin Crosby, 16 Chauncy St., Cambridge, Mass.

BUENOS AIRES—Chairman, G. J. Andrews; Secretary, W. Klappenbach, *La Nacion*, Florida 347, Buenos Aires, Argentina.

BUFFALO-NIAGARA—Chairman, Leroy Fiedler; Secretary, H. G. Korts, 432 Potomac Ave., Buffalo, N. Y.

CHICAGO—Chairman, A. B. Bronwell; Secretary, W. O. Swinyard, Hazeltine Electronics Corp., 325 W. Huron St., Chicago, Ill.

CINCINNATI—Chairman, J. L. Hollis; Secretary, R. S. Butts, 3017 Verdin Ave., Cincinnati 11, Ohio.

CLEVELAND—Chairman, A. S. Nace; Secretary, Lester L. Stoffel, 1095 Kenneth Dr., Lakewood, Ohio

CONNECTICUT VALLEY—Chairman, W. M. Smith; Secretary, R. F. Shea, General Electric Co., Bridgeport, Conn.

DALLAS-FORT WORTH—Chairman, D. J. Tucker; Secretary, P. C. Barnes, WFAA-WBAP, Grapevine, Texas.

DAYTON—Acting Secretary, Joseph General, 1319 Superior Ave., Dayton, 7, Ohio.

DETROIT—Chairman, R. A. Powers; Secretary, R. R. Barnes, 1411 Harvard Ave., Berkley, Mich.

EMPORIUM—Chairman, H. D. Johnson; Secretary, A. Dolnick, Sylvania Electric Products, Inc., Emporium, Pa.

INDIANAPOLIS—Chairman, A. N. Curtiss; Secretary, E. E. Alden, WIRE, Indianapolis, Ind.

KANSAS CITY—Chairman, A. P. Stuhrman; Secretary, R. N. White, 4800 Jefferson St., Kansas City, Mo.

LOS ANGELES—Chairman, L. W. Howard; Secretary, Frederick Ireland, 1000 N. Seward St., Hollywood, 38, Calif.

MONTREAL—Chairman, L. T. Bird; Secretary, J. C. R. Punchard, Northern Electric Co., 1261 Shearer St. Montreal, Que., Canada.

NEW YORK—Chairman, Lloyd Espenschied; Secretary, J. E. Shepherd, 111 Courtenay Rd., Hempstead, L. I., N. Y.

PHILADELPHIA—Chairman, W. P. West; Secretary, S. Gubin, RCA Victor Division, Radio Corporation of America Bldg. 8-10, Camden, N. J.

PITTSBURGH—Chairman, B. R. Teare; Secretary, R. K. Crooks, Box, 2038, Pittsburgh, 30, Pa.

PORTLAND—Chairman, W. A. Cutting; Secretary, W. E. Richardson, 5960 S.W. Brugger, Portland, Ore.

ROCHESTER—Chairman, O. L. Angevine, Jr.; Secretary, G. R. Town, Stromberg-Carlson Co., Rochester, N. Y.

ST. LOUIS—Chairman, N. J. Zehr; Vice Chairman, C. F. Meyer, KFUO, 801 DeMun Ave., St. Louis, Mo.

SAN FRANCISCO—Chairman, W. G. Wagener; Secretary, R. V. Howard, 225 Mallorca Way, San Francisco, Calif.

SEATTLE—Chairman, F. B. Mossman; Secretary, E. H. Smith, Apt. K, 1620—14 Ave., Seattle, 22, Wash.

TORONTO—Chairman, R. G. Anthes; Secretary, J. T. Pfeiffer, Erie Resistor of Canada, Ltd., 128 Peter St., Toronto, Ont., Canada.

TWIN CITIES—Chairman, E. S. Heiser; Secretary, B. R. Hilker, KSTP, St. Paul Hotel, St. Paul, Minn.

WASHINGTON—Chairman, J. D. Wallace; Secretary, F. W. Albertson, c/o Dow and Lohnes, E Street between 13th and 14th Sts., Washington, D. C.